# ECONOMETRICS 227

### DEPARTMENT OF ECONOMICS, UCSD WINTER 2012

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Economics $#210$	Office Hours: Friday afternoons.

#### **Course Outline**

The goal of the class is to familiarize students with Empirical Process Theory and its applications in econometrics. The study of empirical processes will require us to extend basic concepts in statistics and probability to infinite dimensional spaces. As a result, we will need to rely on tools from areas of topology, real analysis and functional analysis. I will try to provide additional background on these topics as necessary, but you should expect a high level of mathematical analysis considerably beyond what is used in the core sequence 220A-220C.

PART I: EMPIRICAL PROCESS THEORY

Math Overview: (i) Measurability; (ii) Topology; (iii) Metric and Hilbert Spaces; (iv) Compactness.

References: Chapters 1.1-1.3 in Durrett (1996), Chapters 2.1-2.4, 2.7-2.10 in Aliprantis and Border (2006), Chapter 2 and 3.1-3.4 in Luenberger (1969).

Weak Convergence: (i) Measurability issues (basic overview); (ii) General theory of Weak Convergence; (iii) Weak Convergence in the Space of Bounded Functions; (iv) Convergence in Outer Probability.

References: Chapter 1.2, 1.3, 1.5, 1.9 and 1.10 in van der Vaart and Wellner (1996).

Empirical Process Theory: (i) Maximal Inequalities; (ii) Symmetrization; (iii) Glivenko-Cantelli Theorems; (iv) Donsker Theorems; (v) Uniform Entropy and Bracketing Numbers; (vi) Permanence of the Donsker Property. *References:* Chapters 2.1-2.7, 2.10 and 2.13 in van der Vaart and Wellner (1996).

PART II: REFINEMENTS AND EXTENSIONS

<u>The Delta Method:</u> (i) Main Result; (ii) Gaussian Limits; (iii) Directional Hadamard differentiability; (iv) Examples. *References:* Chapter 3.9 in van der Vaart and Wellner (1996), Shapiro (1990).

<u>U-Processes:</u> (i) Hoeffding Decomposition; (ii) Degenerate U-Statistics; (iii) Maximal Inequalities. *References:* Chapters 11 and 12 in van der Vaart (1999), Arcones and Gine (1993), Arcones and Gine (1994).

The Bootstrap: (i) Multiplier CLT; (ii) Poissonization; (iii) Empirical Bootstrap; (iv) Delta Method.

References: Chapters 2.9, 3.5, 3.6 and 3.9.4 in van der Vaart and Wellner (1996), Gine and Zinn (1990).

Coupling: (i) Strong Approximations; (ii) Hungarian Construction.

References: Chapter 10 in Pollard (2002), Dudley and Philipp (1983), Komlos et al. (1975), Koltchinskii (1994).

PART III: APPLICATIONS IN ECONOMETRICS

Semiparametric Methods: (i) Simulation estimators; (ii) Cube root asymptotics.

References: Pakes and Pollard (1989), Kim and Pollard (1990).

Nonparametric Methods: (i) Series estimators; (ii) Method of Sieves.

References: Huang (1998), Huang (2003), Chen and Pouzo (2008).

### Web Page and Grading

Course materials will be posted in ted.ucsd.edu. You will be evaluated through problem sets.

### Textbook

There are no required textbooks for the course, although the following will be useful references. If you are planning to specialize in econometric theory you should probably consider adding them to your library.

- 1. "Asymptotic Statistics" by van der Vaart A. W. Cambridge University Press, 1999.
- 2. "Weak Convergence and Empirical Processes" by van der Vaart A. W. and Wellner J. A. Springer, 1996.
- 3. "A User's Guide to Measure Theoretic Probability" by Pollard D. Cambridge University Press, 2002.
- 4. "Introduction to Empirical Processes and Semiparametric Inference" by Kosorok M.R. Springer 2008.

## References

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- CHEN, X. and POUZO, D. (2008). Estimation of nonparametric conditional moment models with possibly nonsmooth moments. Working paper, Yale University.
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