

ECON 281: ALTERNATIVE THEORIES OF CHOICE UNDER UNCERTAINTY

Winter 2013

T,Th 3:30–4:50pm

Econ 304

Prof. Mark Machina

Office: Econ 217

Office Hours: Wed 9:00-1:00

This course examines alternatives to the classic expected utility and subjective expected utility models of risk preferences and beliefs. It begins with an review of the classical models, presents the various systematic violations of them, and then presents and analyzes those alternatives which have been offered.

DATE

TOPIC

Jan. 8	Introduction & Preliminary Concepts
Jan. 10	Expected Utility Risk Preferences under Objective and Subjective Uncertainty
Jan. 15	Evidence on the Underlying Assumptions of the Classical Models
Jan. 17	Evidence on the Underlying Assumptions of the Classical Models (continued)
Jan. 22	Non-Expected Utility Models of Risk Preferences
Jan. 24	Non-Expected Utility Models of Risk Preferences (continued)
Jan. 29	Generalized Expected Utility Analysis
Jan. 31	Generalized Expected Utility Analysis (continued)
Feb. 5	Dynamic Consistency: Arguments and Counterarguments
Feb. 7	Dynamic Consistency: Arguments and Counterarguments (continued)
Feb. 9	Midterm Exam
Feb. 14	Probabilistic Sophistication
Feb. 19	Probabilistic Sophistication (continued)
Feb. 21	Models of Ambiguity and Ambiguity Aversion
Feb. 26	Models of Ambiguity and Ambiguity Aversion (continued)
Feb. 28	Models of Ambiguity and Ambiguity Aversion (continued)
Mar. 5	Subjective Expected Utility Analysis without the Sure-Thing Principle or Probabilistic Beliefs
Mar. 7	Subjective Expected Utility Analysis w/o Sure-Thing Principle or Probabilistic Beliefs (cont.)
Mar. 12	Almost-Objective Uncertainty
Mar. 14	Almost-Objective Uncertainty (continued)
Mar. 19	(Tuesday) FINAL EXAM 3:00-6:00pm
(Review Sessions will be scheduled prior to each exam.)	

READINGS: The readings will consist of handouts, expository articles, classic articles from the literature and current research articles. I will make the required readings available via the web or printed handouts.

EXAMS: The course grade will be determined on the basis of a Midterm and a Final Exam. I will provide practice problems, and there will be review sessions before each exam.

ECONOMICS 281 OUTLINE

Winter 2013

Mark Machina

I. INTRODUCTION & PRELIMINARY CONCEPTS

a. Preliminary Concepts in Probability Theory

Probability Distributions, Probability Measures and Cumulative Distribution Functions
Expected Value, Moments and Stieltjes Integrals
Concave Functions, Convex Functions and Jensen's Inequality
Convolutions, Compound Lotteries and Probability Mixtures

b. The Representation of Uncertainty

Objective Uncertainty: Outcomes, Probabilities and Lotteries
Subjective Uncertainty: States, Events, Acts and Payoff Tables
Mixed Subjective-Objective Uncertainty: "Horse/Roulette Lotteries"
Two-Stage Uncertainty

c. Simple Criteria for Choice Under Uncertainty

First Order Stochastic Dominance Preference
Expected Value Criterion and the St. Petersburg Paradox
Mean-Variance Criterion
Minimax and "Safety-First" Criteria

II. EXPECTED UTILITY RISK PREFERENCES UNDER OBJECTIVE UNCERTAINTY

a. Expected Utility Preferences under Objective Uncertainty

Preferences Functions and von Neumann-Morgenstern Utility Functions
Cardinal and Ordinal Aspects of Utility Functions
Properties of Expected Utility Preferences
The Triangle Diagram

b. Axiomatic Characterization of Expected Utility Preferences

Ordering
Mixture Continuity
The Independence Axiom
Expected Utility Representation Theorem

c. Risk Aversion, Comparative Risk Aversion, Risk Aversion and Wealth

d. Evidence on the Shape of the Utility Function

Friedman-Savage Hypothesis
Skewness Preference, Decreasing Absolute/Increasing Relative Risk Aversion

III. EXPECTED UTILITY RISK PREFERENCES AND BELIEFS UNDER SUBJECTIVE UNCERTAINTY

a. The State-Preference Framework

States, Events, Outcomes and Acts
Preference Functions over Subjective Acts
Objective:Subjective Uncertainty: Anscombe-Aumann Acts

b. Expected Utility Preferences over Subjectively Uncertain Prospects

Subjective Probability and Subjective Expected Utility
Properties of Subjective Expected Utility Preferences
Revealed Comparative Likelihood

- Separability Across Events
 - The Hirshleifer-Yaari Diagram
 - Certainty Line, Fair-Odds Lines and Indifference Curves
 - “Local Risk Neutrality” about Certainty
- c. Risk and Risk Aversion under Subjective Uncertainty**
 - Risk Attitudes in the Hirshleifer-Yaari Diagram
 - Risk Aversion, Risk Preference, and Comparative Risk Aversion
 - Demand for a Risky Asset
 - Risk Aversion and Wealth
- d. Savage’s Joint Characterization of Subjective Probability and Expected Utility**
 - Savage’s Egg Example and Motivation of the Sure-Thing Principle
 - Savage’s Axioms
 - Ordering and Nondegeneracy
 - Eventwise Monotonicity
 - Small Event Continuity
 - Weak Comparative Likelihood
 - Sure Thing Principle
 - Savage’s Proof
- e. Expected Utility Preferences under Mixed Subjective-Objective Uncertainty**
 - Anscombe-Aumann Acts
- f. State-Dependent Expected Utility Preferences**
 - Motivation, Examples, and Applications
 - Violation Comparative Likelihood and Indeterminacy of Beliefs

IV. EVIDENCE ON THE UNDERLYING ASSUMPTIONS OF THE CLASSICAL MODELS

- a. Evidence on the Independence Axiom**
 - “Allais Paradox” and the Common Consequence Effect
 - Common Ratio Effect
 - Oversensitivity to Changes in the Probabilities of Low Probability Events
 - Evidence on Betweenness
- b. Evidence on Transitivity**
 - Threshold and Cyclic Effects
 - The Preference Reversal Phenomenon
- c. Evidence on the Stability of Preferences**
 - Stochastic Choice
 - Invariance of Risk Preferences to Initial Wealth
 - Framing Effects
 - Response Mode Effects and the Preference Reversal Phenomenon
- d. Evidence on the Magnitude of Risk Aversion**
- e. Evidence on the Hypothesis of Probabilistic Sophistication: Ambiguity Aversion**
 - Ellsberg Urns and the Ellsberg Paradoxes
- f. The Evidence from Psychologists**
 - Framing and Reference Point Effects
- g. Validity of the Evidence: Objections and Responses**
- h. Theoretically Induced Violations of Expected Utility**
 - Preferences over Delayed-Resolution Risks
 - Group Risk Preferences

V. NON-EXPECTED UTILITY MODELS OF RISK PREFERENCES

a. Preferences Under Objective Uncertainty

Common Framework: Preference Functionals over Distributions

Key Difference From Expected Utility: Nonlinearity in the Probabilities

b. Separable Functional Form (original version of Prospect Theory)

Criticism of the Separable Form

c. Higher Moments of Utility and General Polynomial Forms

d. Weighted Utility and Karmarker's Model

e. The Rank-Dependent Model

f. The Dual Model

g. Expected Regret/Skew-Symmetric Bilinear Preferences

h. Experimental Tests of Non-Expected Utility Models

VI. GENERALIZED EXPECTED UTILITY ANALYSIS

a. Smooth Preferences and the Local Utility Function

Finite-Outcome Sets: Local Utility Function as a Probability Derivative

Continuum Outcome Sets: Local Utility Function as a Variational Derivative

Local Utility Functions of Some Non-Expected Utility Functional Forms

b. Theoretical Analysis

Robustness of the Classical Analytics

Outcome-Monotonicity and Outcome Derivatives

Characterization of Risk Aversion

Asset Demand and Insurance

Characterization of Comparative Risk Aversion

Comparative Statics

c. Empirical Analysis

Skewness Preference and Hypothesis I

"Fanning Out" and Hypothesis II

Invariance of Gambling Preferences to Initial Wealth

Unbounded Probability Distributions and the St. Petersburg Paradox

d. Applications

Temporal Risk and Induced Preferences

Cooperative Risk Sharing

"Non-Utilitarian" Social Welfare Functions

VII. DYNAMIC CONSISTENCY: ARGUMENTS AND COUNTERARGUMENTS

a. Static, Dynamic and Intertemporal Choice Situations

b. Dynamic Arguments Against Non-Expected Utility Preferences

Argument that Non-Expected Utility Preferences are "Dynamically Inconsistent"

Classical "Making Book" Argument against Non-Expected Utility Preferences

Argument that Non-Expected Utility Maximizers will be "Averse to Information"

c. Hidden Assumption in these Arguments: Consequentialism

d. Consequentialism is Inappropriate when Preferences Are Nonseparable

e. Dynamically Consistent Non-Expected Utility Maximizers

f. Issues in Modeling Nonseparable Preferences under Uncertainty

VIII. PROBABILISTIC SOPHISTICATION

a. Definition of Probabilistic Sophistication

Do the Savage Axioms minus the Sure-Thing Principle imply Prob. Sophistication?

b. Strong Comparative Probability Axiom

Comparison with the Sure-Thing Principle and Weak Comparative Probability Axiom
Ellsberg Urns and the Strong Comparative Probability Axiom

c. Characterization of Probabilistically Sophisticated Non-Expected Utility Preferences

d. Conditional Preferences and Conditional Probability

e. “Minimal” Conditions for Probabilistic Sophistication under Mixed Uncertainty

The Horse-Roulette Replacement Axiom

f. Meaning of “Bayesian Rationality”

IX. MODELS OF AMBIGUITY AND AMBIGUITY AVERSION

a. Maxmin Expected Utility

b. Ordinal Certainty Equivalent Preferences

c. Vector Expected Utility

d. Choquet Expected Utility

e. The “Smooth” Model

f. Variational Preferences

g. Source Preference

h. Critique of the Major Models: The Case of Three or More Outcomes

i. Experimental Tests of Ambiguity Aversion Models

X. SUBJECTIVE EXPECTED UTILITY ANALYSIS WITHOUT THE SURE-THING PRINCIPLE OR PROBABILISTIC SOPHISTICATION

a. Smoothness in the Events, ϕ -Derivatives and Local Event-Additivity

b. Local Probabilistic Sophistication, Local Probabilistic Beliefs and Local Risk Preferences

c. Robustness of the Classical Analytics

Outcome-Monotonicity and Outcome Derivatives

Characterization of Probabilistic Sophistication

Characterization of Comparative and Relative Subjective Likelihood

Characterization of Comparative Risk Aversion under Subjective Uncertainty

XI. ALMOST-OBJECTIVE UNCERTAINTY

a. The Six Properties of Purely Objective Events

b. Almost-Ethically-Neutral Events

c. Almost-Objective Events, Acts and Mixtures

Definition of an Almost-Objective Event

Measure Properties of Almost-Objective Events

Almost-Objective Acts and Mixtures

d. Preferences over Almost-Objective Acts

Revealed Beliefs over Almost-Objective Events

Betting Preferences over Almost-Objective Acts and Mixtures

Objective Risk Preferences Implied by Attitudes Toward Subjective Uncertainty

e. Two Types of Events rather than Two Types of Uncertainty

READINGS (starred readings are required)

The following are some texts and surveys of the material covered in this course

- Abdellaoui, M. and J. Hey (eds.) (2008). *Advances in Decision Making under Risk and Uncertainty*. Dordrecht: Springer.
- Camerer, C. and M. Weber (1992). "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty* 5, 325-370.
- Epstein, L. (1992). "Behavior Under Risk: Recent Developments in Theory and Applications," in Laffont, J.-J. (ed.). *Advances in Economic Theory, Vol. II*. Cambridge University Press.
- Fishburn, P. (1988). *Nonlinear Preference and Utility Theory*. Baltimore: Johns Hopkins Univ. Press.
- Geweke, J. (ed.). (1992). *Decision Making under Risk and Uncertainty: New Models and Empirical Findings*. Dordrecht: Kluwer Academic Publishers.
- Hamouda, O. and J.C.R. Rowley (eds.) (1997b). *Paradoxes, Ambiguity and Rationality* (Vol. 2 of *Foundations of Probability, Econometrics and Economic Games*). Cheltenham, U.K.: Edward Elgar Publishing Ltd.
- Hey, J. and P. Lambert (eds.) (1987). *Surveys in the Economics of Uncertainty*. Oxford: Basil Blackwell Ltd.
- Kelsey, D. and J. Quiggin (1992). "Theories of Choice Under Ignorance and Uncertainty," *Journal of Economic Surveys* 6, 133-153.
- Kischka, P. and C. Puppe (1992). "Decisions under Risk and Uncertainty: A Survey of Recent Developments," *Mathematical Methods of Operations Research* 36, 125-147.
- Machina, M. (1987). "Choice Under Uncertainty: Problems Solved and Unsolved," *Journal of Economic Perspectives*, Summer 1987.
- Machina, M. "Non-Expected Utility Theory," in *The New Palgrave Dictionary of Economics*, 2nd Edition, Steven N. Durlauf and Lawrence E. Blume (eds.), Palgrave Macmillan.
- Munier, B. (1989). "New Models of Decision under Uncertainty: An Interpretive Essay," *European Journal of Operations Research* 38, 307-317.
- Siniscalchi, M. (2008). "Ambiguity and Ambiguity Aversion," in the *New Palgrave Dictionary of Economics*, 2nd Edition, Ed. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan.
- Starmer, C. (2000). "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice Under Risk," *Journal of Economic Literature* 38, 332-382.
- Sugden, R. (1986). "New Developments in the Theory of Choice Under Uncertainty," *Bulletin of Economic Research* 38, 1-24.
- Wakker, P. (2008). "Uncertainty," in the *New Palgrave Dictionary of Economics*, 2nd Edition, Ed. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan.
- Wakker, P. (2010). *Prospect Theory: For Risk and Ambiguity*. Cambridge: Cambridge University Press.
- Weber, M. and C. Camerer (1987). "Recent Developments in Modeling Preferences under Risk," *OR Spektrum* 9, 129-151.

I. INTRODUCTION *The following are some standard on probability. (You presumably know all the probability theory that will be used in this course)*

Billingsley, P. (1986). *Probability and Measure, 2nd Ed.* New York: John Wiley and Sons.

Feller, W. (1968). *An Introduction to Probability Theory and Its Applications, Volume I, 3rd Ed.* New York: John Wiley & Sons.

Feller, W. (1971). *An Introduction to Probability Theory and Its Applications, Volume II, 2nd Ed.* New York: John Wiley & Sons.

Parzen, E. (1960). *Modern Probability Theory and Its Applications.* New York: John Wiley & Sons.

II. EXPECTED UTILITY RISK PREFERENCES UNDER OBJECTIVE UNCERTAINTY

Arrow, K. (1951). "Alternative Approaches to the Theory of Choice in Risk-Taking Situations," *Econometrica* 19, 404-437.

Bernoulli, D. (1738). "Specimen Theoriae Novae de Mensura Sortis," *Commentarii Academiae Scientiarum Imperialis Petropolitanae* [Papers of the Imperial Academy of Sciences in Petersburg] V, 175-192. English translation: "Exposition of a New Theory on the Measurement of Risk," *Econometrica* 22 (1954), 23-36.

Fishburn, P. and P. Wakker (1995). "The Invention of the Independence Condition for Preferences," *Management Science* 41, 1130-1144.

* Friedman, M. and L. Savage (1948). "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* 56, 279-304.

Hammond, P. (1998). "Objective Expected Utility," in Barberá, S., P. Hammond and C. Seidl (1998). *Handbook of Utility Theory. Volume 1: Principles.* Dordrecht: Kluwer Academic Publishers.

* Herstein, I. and J. Milnor (1953). "An Axiomatic Approach to Measurable Utility," *Econometrica* 21, 291-297.

* Machina, M. (1988). "Expected Utility Hypothesis," in John Eatwell, Murray Milgate and Peter Newman, Macmillan (eds.) *The New Palgrave: A Dictionary of Economics.*

Marschak, J. (1950). "Rational Behavior, Uncertain Prospects, and Measurable Utility," *Econometrica* 18, 111-141 ("Errata," *Econometrica* 18, 312).

Pratt, J., H. Raiffa and R. Schlaifer (1964). "The Foundations of Decision Under Uncertainty: An Elementary Exposition," *Journal of the American Statistical Association* 59, 353-375.

Samuelson, P. (1950). "Probability and the Attempts to Measure Utility," *Economic Review* 1, 167-173.

* Samuelson, P. (1952). "Probability, Utility, and the Independence Axiom," *Econometrica* 20, 670-678.

Samuelson, P. (1952). "Utility, Preference, and Probability," (Abstract of a paper given at the conference on "Les Fondements et Applications de la Théorie du Risque en Econométrie," Paris).

Neumann, J. and O. Morgenstern (1944). *Theory of Games and Economic Behavior.* Princeton: Princeton University Press. (2nd Ed. 1947; 3rd Ed. 1953).

III. EXPECTED UTILITY RISK PREFERENCES AND BELIEFS UNDER SUBJECTIVE UNCERTAINTY

- * Anscombe, F. and R. Aumann (1963). "A Definition of Subjective Probability," *Annals of Mathematical Statistics* 34, 199-205.
- Drèze, J. (1974). "Axiomatic Theories of Choice, Cardinal Utility and Subjective Probability," in Drèze, J. (ed.) (1974). *Allocation Under Uncertainty: Equilibrium and Optimality*. London: Macmillan.
- Fishburn, P. (1967). "Preference-Based Definitions of Subjective Probability," *Annals of Mathematical Statistics* 38, 1605-1617.
- Fishburn, P. (1986). "The Axioms of Subjective Probability," *Statistical Science* 1, 335-345.
- Fishburn, P. (1969). "A General Theory of Subjective Probabilities and Expected Utilities," *Annals of Mathematical Statistics* 40, 1419-1429.
- * Hammond, P. (1998). "Subjective Expected Utility," in Barberá, S., P. Hammond and C. Seidl (1998). *Handbook of Utility Theory. Volume 1: Principles*. Dordrecht: Kluwer Academic Publishers.
- * Hirshleifer, J. (1965). "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," *Quarterly Journal of Economics* 79, 509-536.
- Hirshleifer, J. (1966). "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," *Quarterly Journal of Economics* 80, 252-277.
- Karni, E. (1985). *Decision Making Under Uncertainty: The Case of State Dependent Preferences*. Cambridge, Mass.: Harvard University Press.
- Kyburg, H. and H. Smokler (eds.) (1980). *Studies in Subjective Probability, 2nd Ed.* Huntington, New York: Robert E. Krieger Publishing Co.
- Luce, R. and P. Suppes (1965). "Preference, Utility and Subjective Probability," in *Handbook of Mathematical Psychology*, Vol. III, ed. by R. Luce, R. Bush and E. Galanter. New York: John Wiley and Sons.
- Myerson, R. (1979). "An Axiomatic Derivation of Subjective Probability, Utility, and Evaluation Functions," *Theory and Decision* 11, 339-352.
- * Savage, L. (1954). *The Foundations of Statistics*. New York: John Wiley and Sons. Revised and Enlarged Edition, New York: Dover Publications, 1972. [Chapter 2]
- Suppes, P. (1956). "The Role of Subjective Probability and Utility in Decision Making" *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, 1954-1955*, 5, 61-73.

IV. EVIDENCE ON THE UNDERLYING ASSUMPTIONS OF THE CLASSICAL MODELS

- Allais, M. (1953a). "Fondements d'une Théorie Positive des Choix Comportant un Risque et Critique des Postulats et Axiomes de l'Ecole Américaine," *Econométrie, Colloques Internationaux du Centre National de la Recherche Scientifique*, Paris 40, 257-332.
- Allais, M. (1953b). "Le Comportement de l'Homme Rationnel devant le Risque, Critique des Postulats et Axiomes de l'Ecole Américaine," *Econometrica* 21, 503-546. Summarized version of Allais (1953a).

- * Allais, M. (1979). "The Foundations of a Positive Theory of Choice Involving Risk and a Criticism of the Postulates and Axioms of the American School," English translation of Allais (1953a). In Allais, M. and O. Hagen (eds.) (1979). *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht: D. Reidel Publishing Co.
- Allais, M. (1979). "The So-Called Allais Paradox and Rational Decisions Under Uncertainty," in Allais, M. and O. Hagen (eds.) (1979). *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht: D. Reidel Publishing Co.
- Allais, M. (1988). "The General Theory of Random Choices in Relation to the Invariant Cardinal Utility Function and the Specific Probability Function," in Munier, B. (ed.) (1988). *Risk, Decision and Rationality*. Dordrecht: D. Reidel Publishing Co.
- Allais, M. (1994). "An Outline of My Main Contributions to Risk and Utility Theory: Theory, Experience , and Applications," in Munier, B. and M. Machina (eds.) (1994). *Models and Experiments in Risk and Rationality*. Dordrecht: Kluwer Academic Publishers.
- Baron, J. and D. Frisch (1994). "Ambiguous Probabilities and the Paradoxes of Expected Utility," in G. Wright and P. Ayton (eds.), *Subjective Probability*. Chichester, U.K.: Wiley, 273-294.
- Barberá, S. and M. Jackson (1988). "Maximin, Leximin and the Protective Criterion: Characterizations and Comparisons," *Journal of Economic Theory* 46, 34-44.
- * Birnbaum, M. (2008). "New Paradoxes of Risky Decision Making," *Psychological Review* 115, 463-501.
- Birnbaum, M. and A. Chavez (1997). "Tests of Decision Making: Violations of Branch Independence and Distribution Independence," *Organizational Behavior and Human Performance* 71, 161-194.
- * Birnbaum, M. and W. McIntosh (1996). "Violations of Branch Independence in Choices Between Gambles," *Organizational Behavior and Human Performance* 67, 91-110.
- Camerer, C. and T.-H. Ho (1994). "Violations of the Betweenness Axiom and Nonlinearity in Probability," *Journal of Risk and Uncertainty* 8, 167-196.
- Conlisk, J. (1989). "Three Variants on the Allais Example," *American Economic Review* 79, 392-407.
- Curley, S., F. Yates and R. Abrams (1986). "Psychological Sources of Ambiguity Avoidance," *Organizational Behavior and Human Decision Processes* 38, 230-256.
- * Ellsberg, D. (1961). "Risk, Ambiguity and the Savage Axioms," *Quarterly Journal of Economics* 75, 643-669.
- Ellsberg, D. (2001). *Risk, Ambiguity and Decision*. New York: Garland Publishing, Inc.
- Halevy, Y. (2007). "Ellsberg Revisited: An Experimental Study," *Econometrica* 75, 503-536.
- Heath, C. and A. Tversky (1991). "Preferences and Belief: Ambiguity and Competence in Choice Under Uncertainty," *Journal of Risk and Uncertainty* 4, 5-28.
- * Hey, J. and D. di Cagno (1990). "Circles and Triangles: An Experimental Estimation of Indifference Lines in the Marschak-Machina Triangle," *Journal of Behavioral Decision Making* 3, 279-306.
- * Kahneman, D. and A. Tversky (1979). "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica* 47, 263-291.

- * Keller, R. (1985). "The Effects of Problem Representation on the Sure-Thing and Substitution Principles," *Management Science* 31, 738-751.
- Kreps, D. and E. Porteus (1979). "Temporal von Neumann-Morgenstern and Induced Preferences," *Journal of Economic Theory* 20, 81-109.
- * Lichtenstein, S. and P. Slovic (1971). "Reversals of Preferences Between Bids and Choices in Gambling Decisions," *Journal of Experimental Psychology* 89, 46-55.
- Lichtenstein, S. and P. Slovic (1973). "Response-Induced Reversals of Preference in Gambling: An Extended Replication in Las Vegas," *Journal of Experimental Psychology* 101, 16-20.
- * MacCrimmon, K. and S. Larsson (1979). "Utility Theory: Axioms Versus 'Paradoxes,'" in M. Allais and O. Hagen (eds.) (1979). *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht, Holland: D. Reidel Publishing Co.
- * Machina, M. (1987). "Choice Under Uncertainty: Problems Solved and Unsolved," *Journal of Economic Perspectives*, Summer 1987.
- Machina, M. (1983). "Generalized Expected Utility Analysis and the Nature of Observed Violations of the Independence Axiom," in *Foundations of Utility and Risk Theory with Applications*, edited by B. Stigum and F. Wenstøp, D. Reidel, Dordrecht, Holland. Marshall, R., J.-F. Richard and G. Zarkin (1992). "Posterior Probabilities of the Independence Axiom With Nonexperimental Data (or Buckle Up and Fan Out)," *Journal of Business and Economic Statistics* 10, 31-44.
- * Slovic, P. and A. Tversky (1974). "Who Accepts Savage's Axiom?" *Behavioral Science* 19, 368-373.
- Wu, G. and R. Gonzalez (1998). "Common Consequence Conditions in Decision Making Under Risk," *Journal of Risk and Uncertainty* 16, 115-139.

V. NON-EXPECTED UTILITY MODELS OF RISK PREFERENCES

- * Chew, S. (1983). "A Generalization of the Quasilinear Mean With Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox," *Econometrica* 51, 1065-1092.
- Chew, S. and L. Epstein (1989). "A Unifying Approach to Axiomatic Non-Expected Utility Theories," *Journal of Economic Theory* 49, 207-240.
- Chew, S., E. Karni and Z. Safra (1987). "Risk Aversion in the Theory of Expected Utility with Rank Dependent Probabilities," *Journal of Economic Theory* 42, 370-381.
- Dekel, E. (1986). "An Axiomatic Characterization of Preferences Under Uncertainty: Weakening the Independence Axiom," *Journal of Economic Theory* 40, 304-318.
- * Epstein, L. (1992). "Behavior Under Risk: Recent Developments in Theory and Applications," in Laffont, J.-J. (ed.). *Advances in Economic Theory, Vol. II*. Cambridge University Press. Gollier, C. and M. Machina (eds.) (1995). *Non-Expected Utility and Risk Management*, Kluwer Academic Publishers.
- * Fishburn, P. (1984). "SSB Utility Theory and Decision-Making Under Uncertainty," *Mathematical Social Sciences* 8, 253-285.
- Green, J. and B. Jullien (1988). "Ordinal Independence in Non-Linear Utility Theory," *Journal of Risk and Uncertainty* 1, 355-387.
- Gul, F. (1991). "A Theory of Disappointment Aversion," *Econometrica* 59, 667-686.

- Hagen, O. (1979). "Towards a Positive Theory of Preferences Under Risk," in M. Allais and O. Hagen (eds.) *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht, D. Reidel.
- Karmarkar, U. (1978). "Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model," *Organizational Behavior and Human Performance* 21, 61-72.
- Konrad, K. and S. Skaperdas (1993). "Self-Insurance and Self-Protection: A Nonexpected Utility Analysis," *Geneva Papers on Risk and Insurance Theory* 18, 131-146.
- * Loomes, G. and R. Sugden (1982). "Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty," *Economic Journal* 92, 805-824.
- Luce, R. (1991). "Rank- and Sign-Dependent Linear Utility Models for Binary Gambles," *Journal of Economic Theory* 53, 75-100.
- * Machina, M. "Non-Expected Utility Theory," in *The New Palgrave Dictionary of Economics, 2nd Edition*, Steven N. Durlauf and Lawrence E. Blume (eds.), Palgrave Macmillan.
- * Quiggin, J. (1982). "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization* 3, 323-343.
- * Segal U. (1990). "Two-Stage Lotteries without the Reduction Axiom," *Econometrica* 58, 349-377.
- Segal, U. (1988). "Does the Preference Reversal Phenomenon Necessarily Contradict the Independence Axiom?" *American Economic Review* 78, 233-236.
- Sugden, R. (1989). "An Axiomatic Foundation for Regret Theory," manuscript, University of East Anglia.
- Viscusi, W. (1989). "Prospective Reference Theory: Toward an Explanation of the Paradoxes," *Journal of Risk and Uncertainty* 2, 235-264.
- Weil, P. (1990). "Nonexpected Utility in Macroeconomics," *Quarterly Journal of Economics* 105, 29-42.
- * Yaari, M. (1987). "The Dual Theory of Choice Under Risk," *Econometrica* 55, 95-115.

The following debate brought out some particularly evocative language by Professor Allais:

- Machina, M. (1995). "Two Errors in the 'Allais Impossibility Theorem'," *Theory and Decision* 38, 231-250. "The Two Errors: A Summary," *Theory and Decision* 38, 301-307.
- Allais, M. (1995). "The Real Foundations of the Alleged Errors in Allais' Impossibility Theorem: Unceasingly Repeated Errors or Contradictions of Mark Machina," *Theory and Decision* 38, 251-299. "Allais' Rejoinder," *Theory and Decision* 38, 309-311.

Experimental Tests of Non-Expected Utility Models:

- Birnbaum, M. (2004). "Tests of Rank-Dependent Utility and Cumulative Prospect Theory in Gambles Represented by Natural Frequencies: Effects of Format, Event Framing, and Branch Splitting" *Organizational Behavior and Human Decision Processes* 95, 40-65.
- * Camerer, C. (1989). "An Experimental Test of Several Generalized Utility Theories," *Journal of Risk and Uncertainty* 2, 61-104.
- Carbone, E. and J. Hey (1995). "A Comparison of the Estimates of EU and Non-EU Preference Functionals using Data from Pairwise Choice and Complete Ranking Experiments," *Geneva Papers on Risk and Insurance Theory* 20, 111-133.

- Chew, S. and W. Waller (1986). "Empirical Tests of Weighted Utility Theory," *Journal of Mathematical Psychology* 30, 55-72.
- * Harless, D. and C. Camerer (1994). "The Predictive Utility of Generalized Expected Utility Theories," *Econometrica* 62, 1251-1289.
- * Hey, J. and C. Orme (1994). "Investigating Generalizations of Expected Utility Theory Using Experimental Data," *Econometrica* 62, 1291-1326.
- Hey, J. and E. Strazzera (1989). "Estimation of Indifference Curves in the Marschak-Machina Triangle: A Direct Test of the 'Fanning Out' Hypothesis," *Journal of Behavioral Decision Making* 2, 239-260.
- * Starmer, C. (1992). "Testing New Theories of Choice Under Uncertainty Using the Common Consequence Effect," *Review of Economic Studies* 59, 813-830.
- Wu, G. (1994). "An Empirical Test of Ordinal Independence," *Journal of Risk and Uncertainty* 9, 39-60.

VI. GENERALIZED EXPECTED UTILITY ANALYSIS

- * Allen, B. (1987). "Smooth Preferences and the Local Expected Utility Hypothesis," *Journal of Economic Theory* 41, 340-355.
- Bardsley, P. (1993). "Local Utility Functions," *Theory and Decision* 34, 109-118.
- Chew, S., L. Epstein and I. Zilcha (1988). "A Correspondence Theorem Between Expected Utility and Smooth Utility," *Journal of Economic Theory* 46, 186-193.
- Chew, S. and N. Nishimura (1992). "Differentiability, Comparative Statics, and Non-Expected Utility Preferences," *Journal of Economic Theory* 56, 294-312.
- Karni, E. (1987). "Generalized Expected Utility Analysis with State-Dependent Preferences," *International Economic Review* 28, 229-240.
- Karni, E. (1989). "Generalized Expected Utility Analysis of Multivariate Risk Aversion," *International Economic Review* 30, 297-305.
- Karni, E. (1992). "Optimal Insurance: A Nonexpected Utility Analysis," in Dionne, G. (ed.) *Contributions to Insurance Economics*. Boston: Kluwer Academic Publishers.
- * Machina, M. (1982). "'Expected Utility' Analysis Without the Independence Axiom," *Econometrica* 50, 277-323.*
- Machina, M. (1984). "Temporal Risk and the Nature of Induced Preferences," *Journal of Economic Theory* 33, 199-231.
- * Machina, M. (1989). "Comparative Statics and Non-Expected Utility Preferences," *Journal of Economic Theory* 47, 393-405.
- * Machina, M. (1995). "Non-Expected Utility and the Robustness of the Classical Insurance Paradigm," *Geneva Papers on Risk and Insurance Theory* 20, 9-50.
- Wang, T. (1993). " L_p -Fréchet Differentiable Preference and 'Local Utility' Analysis," *Journal of Economic Theory* 61, 139-159.

VII. DYNAMIC CONSISTENCY: ARGUMENTS AND COUNTERARGUMENTS

- Eichberger, J., S. Grant and D. Kelsey (2005). "CEU Preferences and Dynamic Consistency," *Mathematical Social Science* 49, 143-151.
- Eichberger, J. and D. Kelsey (1996). "Uncertainty Aversion and Dynamic Consistency," *International Economic Review* 37, 625-640.
- Epstein, L. and M. Le Breton (1993). "Dynamically Consistent Beliefs Must be Bayesian," *Journal of Economic Theory* 61, 1-22.
- Epstein, L. and S. Zinn (1989). "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica* 57, 937-969.
- Farmer, R. (1990). "RINCE Preferences," *Quarterly Journal of Economics* 105, 43-60.
- Fishburn, P. (1981). "Subjective Expected Utility: A Review of Normative Theories," *Theory and Decision* 13, 139-199.
- Frisch, D. and J. Baron (1988). "Ambiguity and Rationality," *Journal of Behavioral Decision Making* 1, 149-157.
- Green, J. (1987). "'Making Book Against Oneself,' The Independence Axiom and Nonlinear Utility Theory," *Quarterly Journal of Economics* 102, 785-796.
- Hammond, P. (1976). "Changing Tastes and Coherent Dynamic Choice," *Review of Economic Studies* 43, 159-173.
- Hammond, P. (1988). "Consequentialist Foundations for Expected Utility," *Theory and Decision*, 25, 25-78.
- * Hammond, P. (1988). "Consequentialism and the Independence Axiom," in Munier, B. (ed.) (1988). *Risk, Decision and Rationality*. Dordrecht: D. Reidel Publishing Co.
- * Hammond, P. (1989). "Consistent Plans, Consequentialism, and Expected Utility," *Econometrica* 57, 1445-1449.
- Hilton, R. (1990)., "Failure of Blackwell's Theorem Under Machina's Generalization of Expected Utility Analysis without the Independence Axiom," *Journal of Economic Behavior and Organization* 13, 233-244.
- Johnsen, T. and J. Donaldson (1985). "The Structure of Intertemporal Preferences Under Uncertainty and Time Consistent Plans," *Econometrica* 53, 1451-1458.
- Loomes, G. and R. Sugden (1986). "Disappointment and Dynamic Consistency in Choice Under Uncertainty," *Review of Economic Studies* 53, 271-282.
- MacCrimmon, K. (1968). "Descriptive and Normative Implications of the Decision-Theory Postulates," in Borch and Mossin (1968).
- Machina, M. (1981). "'Rational' Decision Making versus 'Rational' Decision Modeling?: A Review of Expected Utility Hypotheses and the Allais Paradox," Edited by Maurice Allais and Ole Hagen," *Journal of Mathematical Psychology* 24, 163-175.
- * Machina, M. (1989). "Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty," *Journal of Economic Literature* 27, 1622-1668.
- Marschak, J. (1951). "Why 'Should' Statisticians and Businessmen Maximize 'Moral Expectation'?" in Neyman, J. (ed.) *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: University of California Press.

- McClennen, E. (1983). "Sure-Thing Doubts," in Stigum, B. and F. Wenstøp (eds.) *Foundations of Utility and Risk Theory with Applications*. D. Reidel Publishing Co.
- McClennen, E. (1990). *Rationality and Dynamic Choice: Foundational Explorations*. Cambridge: Cambridge University Press.
- Morrison, D. (1967). "On the Consistency of Preferences in Allais' Paradox," *Behavioral Science* 12, 373-383.
- Ozdenoren, E. and J. Peck (2008). "Ambiguity Aversion, Games Against Nature, and Dynamic Consistency," *Games and Economic Behavior* 62, 106-115.
- Raiffa, H. (1968). *Decision Analysis: Introductory Lectures on Choices Under Uncertainty*. Reading, Mass.: Addison-Wesley. [Pages 80-86].
- * Samuelson, P. (1988). "How a Certain 'Internal Consistency' Entails the Expected Utility Dogma," *Journal of Risk and Uncertainty* 1, 389-393.
- Segal, U. (1999). "Dynamic Consistency and Non-Expected Utility," in Luini, L. (1999). *Uncertain Decisions: Bridging Theory and Experiments*. Dordrecht: Kluwer Academic Publishers.
- Sugden, R. (1991). "Rational Choice: A Survey of Contributions From Economics and Philosophy," *Economic Journal* 101, 751-785.
- Sobel, J. (1989). "Machina and Raiffa on the Independence Axiom," *Philosophical Studies* 56, 315-329.
- Yaari, M. (1985). "On the Role of 'Dutch Books' in the Theory of Choice Under Risk," Nancy Schwartz Memorial Lecture, Graduate School of Management, Northwestern University.

VIII. PROBABILISTIC SOPHISTICATION

- Epstein, L. and M. Le Breton (1993). "Dynamically Consistent Beliefs Must be Bayesian," *Journal of Economic Theory* 61, 1-22.
- Grant, S. (1995). "Subjective Probability Without Monotonicity: Or How Machina's Mom May Also Be Probabilistically Sophisticated," *Econometrica* 63, 159-189.
- * Machina, M. and D. Schmeidler (1992). "A More Robust Definition of Subjective Probability," *Econometrica* 60, 745-780.
- * Machina, M. and D. Schmeidler (1995). "Bayes without Bernoulli: Simple Conditions for Probabilistic Choice," *Journal of Economic Theory* 67, 106-128.

IX. MODELS OF AMBIGUITY AND AMBIGUITY AVERSION

- Bewley, T. (1986). "Knightian Decision Theory: Part I," Yale University, Cowles Foundation Discussion Paper No. 807.
- Bewley, T. (1987). "Knightian Decision Theory: Part II: Intertemporal Problems," Yale University, Cowles Foundation Discussion Paper No. 835.
- Bewley, T. (1988). "Knightian Decision Theory and Econometric Inference," Yale University, Cowles Foundation Discussion Paper No. 868.
- Bewley, T. (1989). "Market Innovation and Entrepreneurship: A Knightian View," Yale University, Cowles Foundation Discussion Paper No. 905.

- Brewer, K. and W. Fellner (1965). "The Slanting of Subjective Probabilities: Agreement on Some Essentials," *Quarterly Journal of Economics* 79, 657-663.
- * Camerer, C. and M. Weber (1992). "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty* 5, 325-370.
- Cohen, M. and J.-Y. Jaffray (1980). "Rational Behavior Under Complete Ignorance," *Econometrica* 48, 1281-1299.
- Epstein, L. (1999). "A Definition of Uncertainty Aversion," *Review of Economic Studies* 66, 579-608.
- Epstein, L. and J. Zhang (2001). "Subjective Probabilities on Subjectively Unambiguous Events," *Econometrica* 69, 265-306.
- * Ergin, H. and F. Gul (2009). "A Theory of Subjective Compound Lotteries," *Journal of Economic Theory* 144, 899-929.
- Fellner, W. (1961). "Distortion of Subjective Probabilities as a Reaction to Uncertainty," *Quarterly Journal of Economics* 75, 670-689.
- Fishburn, P. (1989). "Nontransitive Measurable Utility for Decision Under Uncertainty," *Journal of Mathematical Economics* 18, 187-207.
- Fishburn, P. (1993). "The Axioms and Algebra of Ambiguity," *Theory and Decision* 34, 119-137.
- Ghirardato, P. and M. Marinacci (1999). "Ambiguity Made Precise: A Comparative Foundation," Social Science Working Paper 1026, California Institute of Technology.
- Ghirardato, P., F. Maccheroni, M. Marinacci and M. Siniscalchi (2003). "A Subjective Spin on Roulette Wheels," *Econometrica* 71, 1897-1908.
- Ghirardato, P., F. Maccheroni and M. Marinacci (2004). "Differentiating Ambiguity and Ambiguity Attitude," *Journal of Economic Theory* 118, 133-173.
- Ghirardato, P. and M. Marinacci (2002). "Ambiguity Made Precise: A Comparative Foundation," *Journal of Economic Theory* 102, 251-289.
- * Gilboa, I. (1987). "Expected Utility with Purely Subjective Non-Additive Probabilities," *Journal of Mathematical Economics* 16, 65-88.
- * Gilboa, I. and D. Schmeidler (1989). "Maximin Expected Utility With a Non-Unique Prior," *Journal of Mathematical Economics* 18, 141-153.
- Hogarth, R. and H. Kunreuther (1985). "Ambiguity and Insurance Decisions," *American Economic Review* 75, 386-390.
- Kahn, B. and Sarin, R. (1988). "Modeling Ambiguity in Decisions Under Uncertainty," *Journal of Consumer Research* 15, 265-272.
- Karni, E. and D. Schmeidler (1990). "Utility Theory with Uncertainty," in *Handbook of Mathematical Economics, Vol. 4*, ed. by W. Hildenbrand and H. Sonnenschein. Amsterdam: North-Holland Publishing Co.
- Kelsey, D. and F. Milne (1996). "Induced Preferences, Non Additive Probabilities and Multiple Priors," manuscript, Dept. of Economics, University of Birmingham.
- * Kelsey, D. and J. Quiggin (1992). "Theories of Choice Under Ignorance and Uncertainty," *Journal of Economic Surveys* 6, 133-153.

- * Maccheroni F., M. Marinacci and A. Rustichini (2006). "Ambiguity Aversion, Robustness, and Variational Representation of Preferences" *Econometrica* 74, 1447-1498.
- Maskin, E. (1979). "Decision Making Under Ignorance with Implications for Social Choice," *Theory and Decision* 11, 319-337.
- Montesano, A. and F. Giovannoni (1996). "Uncertainty Aversion and Aversion to Increasing Uncertainty," *Theory and Decision* 41, 133-148.
- Nau, R. (1995). "Coherent Decision Analysis with Inseparable Probabilities and Utilities," *Journal of Risk and Uncertainty* 10, 71-91.
- Nau, R. (2001). "A Generalization of the Pratt-Arrow Measure to Non-Expected-Utility Preferences and Inseparable Probability and Utility," manuscript, Duke University.
- Sarin, R. and P. Wakker (1998). "Revealed Likelihood and Knightian Uncertainty," *Journal of Risk and Uncertainty* 16, 223-250.
- * Schmeidler, D. (1989). "Subjective Probability and Expected Utility without Additivity," *Econometrica* 57, 571-587.
- * Segal, U. (1987). "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach," *International Economic Review* 28, 175-202.
- Selden, L. (1978). "A New Representation of Preferences over 'Certain \times Uncertain' Consumption Pairs: The 'Ordinal Certainty Equivalent' Hypothesis," *Econometrica* 46, 1045-1060.
- Sinn, H.-W. (1980). "A Rehabilitation of the Principle of Insufficient Reason," *Quarterly Journal of Economics* 94, 493-506.
- Siniscalchi, M. (2008). "Ambiguity and Ambiguity Aversion," in the *New Palgrave Dictionary of Economics*, 2nd Edition, Ed. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan.
- Tversky, A. and D. Koehler (1994). "Support Theory: A Nonextensional Representation of Subjective Probability," *Psychological Review* 101, 547-567.
- * Wakker, P. (2008). "Uncertainty," in *The New Palgrave Dictionary of Economics*, 2nd Edition, Steven N. Durlauf and Lawrence E. Blume (eds.), Palgrave Macmillan.
- Wakker, P., I. Erev and E. Weber (1994). "Comonotonic Independence: The Critical Test Between Classical and Rank-Dependent Utility Theories," *Journal of Risk and Uncertainty* 9, 195-230.
- Zhang, J. (2002). "Subjective Ambiguity, Expected Utility and Choquet Expected Utility," *Economic Theory* 20, 159-181.

Experimental Tests of Ambiguity Aversion Models:

- * Baillon A., O. L'Haridon and L. Placido (2011). "Ambiguity Models and the Machina Paradoxes" *American Economic Review* 101, 1547-1560.
- Cohen, M., I. Gilboa, J.-Y. Jaffray and D. Schmeidler (2000). "An Experimental Study of Updating Ambiguous Beliefs," *Risk, Decision and Policy* 5, 123-133.
- * Curley, S. and F. Yates (1989). "An Empirical Evaluation of Descriptive Models of Ambiguity Reactions in Choice Situations," *Journal of Mathematical Psychology* 33, 397-427.
- di Mauro, C. and A. Maffioletti (1996). "An Experimental Investigation of the Impact of Ambiguity on the Valuation of Self-Insurance and Self-Protection," *Journal of Risk and Uncertainty* 13, 53-71.

- Fennema, H. and P. Wakker (1996). "A Test of Rank-Dependent Utility in the Context of Ambiguity," *Journal of Risk and Uncertainty* 13, 19-35.
- Ford J. and S. Ghose (1998). "Ellsberg's Urns, Ambiguity, Measures of Uncertainty and Non-Additivity: Some Experimental Evidence," *Applied Economics Letters* 5, 147-151.
- * Hey, J., G. Lotito and A. Maffioletti (2010). "The Descriptive and Predictive Adequacy of Theories of Decision Making under Uncertainty/Ambiguity," *Journal of Risk and Uncertainty* 41, 81-111.
 - * L'Haridon, O. and L. Placido (2010). "Betting on Machina's Reflection Example: An Experiment on Ambiguity," *Theory and Decision* 69, 375-393.
- Machina, M. (2009). "Risk, Ambiguity, and the Rank-Dependence Axioms," *American Economic Review* 99, 385-392.
- * Machina, M. (2012). "Ambiguity Aversion with Three or More Outcomes," manuscript, UCSD.
 - * Manglesdorff, L. and M. Weber (1994). "Testing Choquet Expected Utility," *Journal of Economic Behavior and Organization* 25, 437-457.
- Marinacci, M. (2002). "Learning From Ambiguous Urns," *Statistical Papers* 43 143-151.
- Yang, C.-L. and L. Yao (2011). "Testing Ambiguity Theories: Revisiting Ellsberg's Paradox in a New Experimental Design," manuscript, Academia Sinica, Taipei.

X. SUBJECTIVE EXPECTED UTILITY ANALYSIS WITHOUT THE SURE-THING PRINCIPLE OR PROBABILISTIC SOPHISTICATION

- Epstein, L. (1999). "A Definition of Uncertainty Aversion," *Review of Economic Studies* 66, 579-608.
- Epstein, L. and J. Zhang (1999). "Least Convex Capacities," *Economic Theory* 13, 263-286.
- * Machina, M. (2005). "'Expected Utility/Subjective Probability' Analysis without the Sure-Thing Principle or Probabilistic Sophistication," *Economic Theory* 26, 1-62.

XI. ALMOST-OBJECTIVE UNCERTAINTY

- * Machina, M. (2004). "Almost-Objective Uncertainty," *Economic Theory* 24, 1-54.
 - * Quiggin, J. (2007). "Ambiguity and the Value of Information: An Almost-Objective Events Analysis," *Economic Theory* 30, 409-414.
- Sagi, J. (2007). "Modeling Implications of Source-Invariance to Machina's 'Almost Objective' Fair Bets," mimeo, Vanderbilt University.