# Econ 205: Mathematics Overview 

Fall 2006, Prof. Joel Watson

This course is a rapid review of topics in analysis, calculus, and optimization. The main objectives of the course are to (i) expose students to the basic mathematical definitions and methods that are used in the Ph.D. core sequences and (ii) stimulate interaction between the firstyear class, in particular in fostering study groups that will be useful in the core courses. Some students will find the course challenging, while some with recent mathematics training will find much of the material review. Everyone should recognize the importance of ongoing mathematical study.

Schedule: 8:30-11:00 a.m. daily (Monday through Friday) from Wednesday, August 23, through Monday, September 18. Class will not be held on the following dates: Monday, September 4 (Labor Day); Monday, September 11 (International Center orientation 9:00-4:00); and Thursday, September 14 (Office of Graduate Studies orientation 8:30-4:40). There will also be a problemsolving and help session every day at 3:00 p.m; this session may also be occasionally used for the presentation of optional material.

Exams: The final exam will take place on Monday, September 18. There will also be a few graded quizzes.

Problem Sets: Working through exercises is an important part of learning mathematics. The course outline includes a list of exercises from the textbooks; supplemental problems will also be provided. Students should work diligently on them. This work will neither be collected nor graded.

Grading: All entering Ph.D. students are required to pass this course. Each student's course grade will be computed as the maximum of his/her final exam grade, and a weighted average of the final exam grade $(75 \%)$ and grades on quizzes. Students with very good training in mathematics are allowed to skip the lectures, but everyone must take the final examination.

Textbooks and Reference Material: The following textbook is available at the bookstore:
(SB) Simon and Blume, Mathematics for Economists (Norton).
Having access to this or a comparable textbook is recommended. Students are welcome to use any other suitable book as a primary reference. Other textbooks that cover some of the course material are:

Binmore, Calculus and Mathematical Analysis (Cambridge)
Chiang, Fundamental Methods of Mathematical Economics,
Dixit, A., Optimization in Economic Theory (Oxford),
Intriligator, M., Mathematical Optimization and Economic Theory (Prentice-Hall),
(JP) Johnsonbaugh and Pfaffenberger, Foundations of Mathematical Analysis,
Marsden and Tromba, Vector Calculus, and
(N) Novshek, Mathematics for Economists.

| Topic | Reading | Exercises |
| :---: | :---: | :---: |
| The Real Line |  |  |
| 1. Basic set theory, functions, real numbers, properties (intervals, bounds, inf, sup, min, max) | $\begin{aligned} & \text { JP } 1,3,5 \\ & \text { N } 1 \\ & \text { SB 1, 2, A1 } \end{aligned}$ | $\begin{aligned} & 1.1-1.5,5.1-5.5 \\ & 2.4,4.1,4.2 \end{aligned}$ |
| 2. Mathematical induction; methods of proof | $\begin{aligned} & \text { JP 6, } 7 \\ & \text { SB A1 } \end{aligned}$ | 6.1, 6.2, 6.5, 7.3 |
| 3. Sequences, subsequences, convergence | $\begin{aligned} & \text { JP 10-14, } \\ & 16 \text { (to p50) } \\ & \text { N } 3 \\ & \text { SB } 29 \end{aligned}$ | $\begin{aligned} & 10.1-10.4,10.9,10.11,11.9 \\ & 12.1-12.4,13.1,13.3,14.1 \end{aligned}$ |
| 4. Bolzano-Weierstrass theorem, Cauchy condition | $\begin{aligned} & \text { JP 18, } 19 \\ & \text { SB } 29 \end{aligned}$ | 18.1, 18.2, 18.5, 19.1, 19.4 |
| 5. Functions, limits of functions, continuity | $\begin{aligned} & \text { JP 2, 30-33 } \\ & \text { N 1 } \\ & \text { SB } 2 \end{aligned}$ | $30.1,30.2,30.8,31.3,32.5,33.1,33.2$ |
| 6. Differentiation (continuity, chain rule, l'Hopital's rule) | $\begin{aligned} & \text { JP 48, } 49 \\ & \text { N } 1 \\ & \text { SB 2, } 4 \end{aligned}$ | $\begin{aligned} & 48.1,48.2,49.1,49.2 \\ & 2.7-2.9,211,2.12,2.15,2.16,3.1,4.4-4.6 \end{aligned}$ |
| 7. Mean-value theorems and Taylor's theorem | $\begin{aligned} & \text { JP } 49,50 \\ & \text { N } 1 \\ & \text { SB } 30 \end{aligned}$ | $\begin{aligned} & 50.1,50.2,50.4 \\ & 2.1,30.7,30.8 \end{aligned}$ |
| 8. Univariate optimization, concavity, convexity (first- and second-order conditions) | $\begin{aligned} & \text { N } 1 \\ & \text { SB } 3 \end{aligned}$ | 3.4, 3.5, 3.11 |
| 9. Integration | $\begin{aligned} & \text { (JP 51-58) } \\ & \text { N } 1 \\ & \text { SB A4 } \end{aligned}$ | $\begin{aligned} & 1.1-3 \\ & \mathrm{~A} 4.1-\mathrm{A} 4.4 \end{aligned}$ |
| Euclidean Space and Vector Calculus |  |  |
| 10. Concepts of Euclidean space (vectors, matrices, geometry of real-valued functions, metrics, open/closed/compact sets, continuity, eigenvalues/vectors) | N 2,3 <br> SB 10, 12, 23 | $\begin{aligned} & 3.1,3.4,3.6-13 \\ & 10.1-10.3,10.11-10.13 \\ & 10.27-10.31,10.32,10.38-10.40 \\ & 23.1-23.5,23.47-23.52 \end{aligned}$ |
| 11. Differentiation (gradient, continuity, chain rule, iterated partials) | N 5 <br> SB 13, 14 | $\begin{aligned} & 5.1-7 \\ & 13.17,13.21,14.1,14.2,14.11,14.13 \text {, } \\ & 14.18,14.20-14.22 \end{aligned}$ |
| 12. Taylor's theorem | N 5 <br> SB 14, 30 | 14.24.14.28, 30.13, 30.14 |
| 13. Unconstrained optimization, concavity, convexity (first- and second-order conditions) | N 5 <br> SB 16, 17, 30 | 17.1, 17.2 |
| 14. Implicit function theorem, envelope theorem | $\begin{aligned} & \text { N } 7,8 \\ & \text { SB } 15 \end{aligned}$ | $\begin{aligned} & 8.2-5,8.8,8.10 \\ & 15.6,15.8,15.13,15.18,15.21,15.22 \end{aligned}$ |
| 15. Overview of Constrained optimization (quasiconcavity, first-order conditions) | N 5 <br> SB 18,19 | $\begin{aligned} & \text { 6.1-36 } \\ & 18.2-18.7,18.10-18.12,18.15,19.3,19.4 \end{aligned}$ |
| 16. Overview of Dynamic Optimization (continuous or discrete) |  |  |
| 17. Overview of differential equations | SB 24, 25 |  |
| 18. Lead-in for the core sequences (examples) |  |  |

