ECON 210D; SYLLABUS

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Purpose:

In this course we will study techniques used to analyze dynamic macro- economic models. These models distinguish themselves from many other economic and numeric problems in that the solutions are functions. In this research area, therefore, one has to rely on function spaces and on numerical techniques that can handle compositions of functions. The first part of this course focuses on the theory of dynamic optimization theory. The second part of this course focuses on techniques to obtain numerical solutions. I will use examples from the growth literature to illustrate the techniques. Several of the numerical techniques discussed are useful in other areas of economic research as well (For example, "approximation methods" and "numerical integration methods" are used in econometrics.)

Course Outline

1. Useful Numerical Techniques (4 lectures)

- Numerical Optimization.
- Nonlinear Equation Solvers.
- Approximation Methods.
- Numerical Integration Methods.

We will spend most of our time on Approximation Methods (Local Approximation, Uniform Approximation, Orthogonal Polynomials, Splines, Finite Element Approaches, & Neural Networks) and Numerical Integration Methods (Newton-Cotes Formulas, Gaussian Quadrature Formulas, Monte-Carlo and Numerical Integration).

2. Introduction to Dynamic Optimization Problems (2 lectures).

- Calculus of Variation
- Optimal Control.
- Dynamic Programming.
- Application: The Solow-Swann growth model and the Ramsey-Cass- Koopmans growth model.

In this part of the course, we will focus on the derivation of the first-order conditions and discuss the differences between calculus of variation, optimal control, and dynamic programming. Although most of the discussion in this course focuses on discrete time problems, we will use the continuous time approach to compare the Solow-Swann growth model with the Ramsey model.

3. Dynamic Programming (4 lectures).

- Mathematical Preliminaries.
- Contraction Mapping Theorem.
- Existence.
- Continuity and Differentiability.
- Application: Exogenous growth, Learning by Doing, Human Capital Accumulation, and the AK model.

4. Linear Systems (2 lectures).

- Stability.
- Multiple Solutions (Sun spots and self-fulfilling hypotheses).
- Convergence.
- Applications: Barro growth regressions, convergence, externalities.

Powerful analytic tools are available to study questions like uniqueness and stability in linear systems. We will show that these questions are related. That is, the more stability in the system, the more likely it is that there are multiple solutions including sun spots.

5. The Mechanics of Popular Solution Algorithms (2 lectures).

- Linear Quadratic (LQ).
- Value Function Iteration.
- Linearized First-Order Conditions.
- Parameterized Expectations Algorithm (PEA).
- Policy Function Iteration.

In this section we will describe how to implement five popular solution methods.

6. General Treatment of Solution Algorithms (4 lectures).

- Projection Methods.
- Perturbation Methods.

Solving dynamic macro models often requires dealing with compositions of functions. Simple numerical techniques to solve difference or differential equations like "finite difference methods", therefore, seldom can be used. We will develop a general framework to obtain global approximations (projection methods). Most algorithms used in the literature can be viewed as a special case, i.e., as a projection method using a particular choice of approximation function, a particular choice of elements in the state space to solve for the parameters, and a particular choice of weighting the approximation errors in case there are more equations than parameters. Perturbation methods are local approximation methods, but the higher-order versions often have good global approximation properties. These methods are rarely used in economics but are likely to become more popular when models with high-dimensional state spaces have to be solved.

Course Requirements:

There are two requirements. The first is to take the five quizzes and the second is to complete a series of numerical projects assigned during the course. The course grade is a weighted average of the grade of the quizzes (45%), the grade of the projects (45%), and class participation (10%).

Highly recommended Reading Material:

- Judd, K.L., 1991, Numerical Methods in Economics, Chapters 2, 6, 7, 11, 12, and 13.
- Stokey, N.L. , R.E. Lucas Jr. , with E.C. Prescott, 1989, Recursive Methods in Economic Dynamics, Chapters 1-4.

Related Literature.

Dynamic Optimization.
- Intrilligator, M.D., 1971, Mathematical Optimization and Economic Theory.
- Kamien, M.I., and N.L. Schwartz, 1991, Dynamic Optimization.
- Frontiers in Business Cycle Research, T.F. Cooley (ed.).

Numerical Methods. There are many good textbooks on numerical methods. I have found Numerical Recipes often very useful to get a quick idea about the theoretical issues involved and the available algorithms.

Solution Methods. The 1990 January issue of the Journal of Business and Economic Statistics is a useful special issue on numerical solution techniques including value function iteration, linearization, PEA, and policy function iteration, among others. Other useful papers are
- Christiano, L.J. and J.D.M. Fisher, 1994, Algorithms for Solving Dynamic Models with Occasionally Binding Constraints, Federal Reserve Bank of Chicago Working Paper Series WP-94-6.
- Den Haan, W.J., 1991, Introduccion a los metodos numericos para resolver modelos dinamicos estocasticos (An introduction to numerical methods to solve stochastic dynamic models), Cuadernos Economicos, Numero 48 2, translated into Spanish.
- Den Haan, W.J., 1996a, Understanding Equilibrium Models with a Small and a Large Number of Agents. (On Solving Heterogeneous Agent Models).
- Judd, K, 1992, Projection Methods for Solving Economic Growth Models, Journal of Economic Theory.
- King, R.G., C. Plosser, and S. Rebelo, 1988, Production, Growth and Business Cycles, 1. The basic neoclassical model, Journal of Monetary Economics 21, 195-232. (On linearization).

Economic Growth:
- Barro, R., and X. Sala-i-Martin, 1993, Economic Growth, Chapters 1-4.
- Romer, D., 1996, Advanced Macroeconomics, Chapters 1-3.