Lecture 10
Further clarifications, common errors, and examples

1. Negation
2. Statement variables
3. Operator specificity
4. Component order
5. Subproof requirements
6. Examples

Negation

The appearance of a tilde in the inference or replacement rules only indicates that one statement is the negation of another, not that it has to have a tilde in front of it.

$\varphi \supset \psi$	x. $(P \lor Q) \supset {\sim}T$	
φ	y. $(P \lor Q)$	
$\therefore \psi$	z. ${\sim}T$	x, y MP

Negation

$\varphi \supset \psi$	x. $(P \lor Q) \supset {\sim}T$	
${\sim}\psi$	y. T	
$\therefore {\sim}\varphi$	z. ${\sim}(P \lor Q)$	x, y MT

Negation

$${\sim}(\varphi \lor \psi) \;::\; {\sim}\varphi \bullet {\sim}\psi$$

x. ${\sim}(P \lor Q)$
y. ${\sim}P \bullet {\sim}Q$

x. ${\sim}P \bullet {\sim}Q$
y. ${\sim}(P \lor Q)$

x. $P \bullet Q$
y. ${\sim}({\sim}P \lor {\sim}Q)$

Statement variables

Greek letters are statement variables. They stand for any statement, simple or compound. If the same variable shows up more than once, this implies that the statement must be the same.

But, different variable letters don't require that the statements be different.

Statement variables

$\varphi \supset \psi$	x. ${\sim}(P \bullet Q) \supset Z$
$\psi \supset \chi$	y. $Z \supset (R \lor S)$
$\therefore \varphi \supset \chi$	z. ${\sim}(P \bullet Q) \supset (R \lor S)$

~~x. ${\sim}(P \bullet Q) \supset Z$~~
~~y. $Z \supset (R \lor S)$~~
~~z. ${\sim}(P \bullet Q) \supset R$~~

x. $(R \lor S) \supset Z$
y. $Z \supset (R \lor S)$
z. $(R \lor S) \supset (R \lor S)$

Operator specificity

The opertors specified in the inference and replacement rules are not substitutable (except negation, as noted above).

$\varphi \supset \psi$
φ
$\therefore \psi$

x. (P v Q) v ~T
y. (P v Q)
z. ~T x, y MP

Operator specificity

$\sim(\varphi \vee \psi) :: \sim\varphi \bullet \sim\psi$

x. ~(P ⊃ Q)
y. ~P • ~Q x DeM

Component order

The disjunction, conjunction, and biconditional operators are order invariant, so the order of disjuncts, conjuncs, and bicoonditional components as they appear in the rules does not matter.

Conditionals are not order invariant, so their order matters.

Component order

$\varphi \vee \psi$
$\sim\varphi$
$\therefore \psi$

x. P v ~(H ⊃ K)
y. ~P
z. ~(H ⊃ K)

x. P v ~(H ⊃ K)
y. H ⊃ K
z. P

Component order

$\varphi \bullet \psi$
$\therefore \varphi$

x. (D v B) • ~(H ⊃ K)
y. D v B

x. (D v B) • ~(H ⊃ K)
y. ~(H ⊃ K)

Component order

$\varphi \supset \psi$
$\sim\psi$
$\therefore \sim\varphi$

x. (D v B) ⊃ ~(H ⊃ K)
y. H ⊃ K
z. ~(D v B) x, y MT

x. (D v B) ⊃ ~(H ⊃ K)
y. ~(D v B)
z. H ⊃ K x, y MT

Subproof requirements

When using subproofs, recall that a proof cannot end until all subproofs are closed.

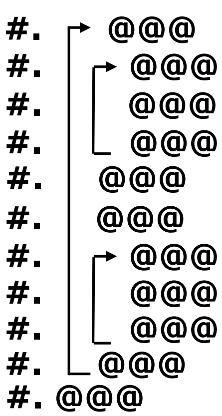
Graphically, this means that there won't be any indenting arrows unconnected to verrtical lines.

Subproof requirements

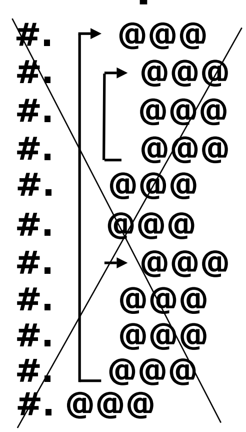
If at any point in a proof there is more than on discharged assumption, the most recent undischarged assumption must be discharged first.

Graphically, this means that lines used to cloose subproofs will never cross.

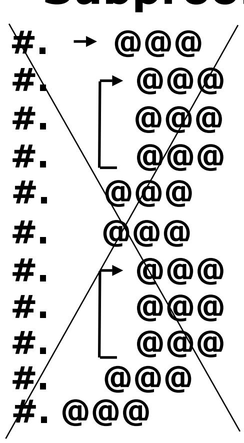
Subproof requirements



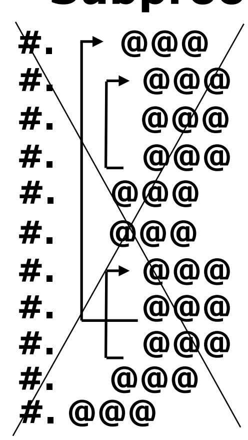
Subproof requirements



Subproof requirements



Subproof requirements



Subproof requirements

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01. (R v S) ⊃ T
02. (P v Q) ⊃ T
03. R v P / ∴ T

n-8. R v P
n-7. (R v P) v (S v Q) n-8 DI
n-6. {(R v P) v S} v Q n-7 assoc
n-5. {R v (P v S)} v Q n-6 assoc
n-4. {R v (S v P)} v Q n-5 comm
n-3. {(R v S) v P} v Q n-4 assoc
n-2. (R v S) v (P v Q) n-3 assoc
n-1. T v T n-2, 1, 2 dil
n. T n-1 dup

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01. (R v S) ⊃ T
02. (P v Q) ⊃ T
03. R v P / ∴ T
04. (R v P) v (S v Q) 3 DI

n-7. (R v P) v (S v Q) n-8 DI
n-6. {(R v P) v S} v Q n-7 assoc
n-5. {R v (P v S)} v Q n-6 assoc
n-4. {R v (S v P)} v Q n-5 comm
n-3. {(R v S) v P} v Q n-4 assoc
n-2. (R v S) v (P v Q) n-3 assoc
n-1. T v T n-2, 1, 2 dil
n. T n-1 dup

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01. (R v S) ⊃ T
02. (P v Q) ⊃ T
03. R v P / ∴ T
04. (R v P) v (S v Q) 3 DI
05. {(R v P) v S} v Q 4 assoc

n-6. {(R v P) v S} v Q n-7 assoc
n-5. {R v (P v S)} v Q n-6 assoc
n-4. {R v (S v P)} v Q n-5 comm
n-3. {(R v S) v P} v Q n-4 assoc
n-2. (R v S) v (P v Q) n-3 assoc
n-1. T v T n-2, 1, 2 dil
n. T n-1 dup

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01. (R v S) ⊃ T
02. (P v Q) ⊃ T
03. R v P / ∴ T
04. (R v P) v (S v Q) 3 DI
05. {(R v P) v S} v Q 4 assoc
06. {R v (P v S)} v Q 5 assoc

n-5. {R v (P v S)} v Q n-6 assoc
n-4. {R v (S v P)} v Q n-5 comm
n-3. {(R v S) v P} v Q n-4 assoc
n-2. (R v S) v (P v Q) n-3 assoc
n-1. T v T n-2, 1, 2 dil
n. T n-1 dup

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01. (R v S) ⊃ T
02. (P v Q) ⊃ T
03. R v P / ∴ T
04. (R v P) v (S v Q) 3 DI
05. {(R v P) v S} v Q 4 assoc
06. {R v (P v S)} v Q 5 assoc
07. {R v (S v P)} v Q 6 comm

n-4. {R v (S v P)} v Q n-5 comm
n-3. {(R v S) v P} v Q n-4 assoc
n-2. (R v S) v (P v Q) n-3 assoc
n-1. T v T n-2, 1, 2 dil
n. T n-1 dup

01.	(R v S) ⊃ T	
02.	(P v Q) ⊃ T	
03.	R v P	/ ∴ T
04.	(R v P) v (S v Q)	3 DI
05.	{(R v P) v S} v Q	4 assoc
06.	{R v (P v S)} v Q	5 assoc
07.	{R v (S v P)} v Q	6 comm
08.	{(R v S) v P} v Q	7 assoc
n-3.	{(R v S) v P} v Q	n-4 assoc
n-2.	(R v S) v (P v Q)	n-3 assoc
n-1.	T v T	n-2, 1, 2 dil
n.	T	n-1 dup

01.	(R v S) ⊃ T	
02.	(P v Q) ⊃ T	
03.	R v P	/ ∴ T
04.	(R v P) v (S v Q)	3 DI
05.	{(R v P) v S} v Q	4 assoc
06.	{R v (P v S)} v Q	5 assoc
07.	{R v (S v P)} v Q	6 comm
08.	{(R v S) v P} v Q	7 assoc
09.	(R v S) v (P v Q)	8 assoc
n-2.	(R v S) v (P v Q)	n-3 assoc
n-1.	T v T	n-2, 1, 2 dil
n.	T	n-1 dup

01.	(R v S) ⊃ T	
02.	(P v Q) ⊃ T	
03.	R v P	/ ∴ T
04.	(R v P) v (S v Q)	3 DI
05.	{(R v P) v S} v Q	4 assoc
06.	{R v (P v S)} v Q	5 assoc
07.	{R v (S v P)} v Q	6 comm
08.	{(R v S) v P} v Q	7 assoc
09.	(R v S) v (P v Q)	8 assoc
10.	T v T	9, 1, 2 dil
n-1.	T v T	n-2, 1, 2 dil
n.	T	n-1 dup

01.	(R v S) ⊃ T	
02.	(P v Q) ⊃ T	
03.	R v P	/ ∴ T
04.	(R v P) v (S v Q)	3 DI
05.	{(R v P) v S} v Q	4 assoc
06.	{R v (P v S)} v Q	5 assoc
07.	{R v (S v P)} v Q	6 comm
08.	{(R v S) v P} v Q	7 assoc
09.	(R v S) v (P v Q)	8 assoc
10.	T v T	9, 1, 2 dil
11.	T	10 dup
n.	T	n-1 dup

01.	(R v S) ⊃ T	
02.	(P v Q) ⊃ T	
03.	R v P	/ ∴ T
04.	(R v P) v (S v Q)	3 DI
05.	{(R v P) v S} v Q	4 assoc
06.	{R v (P v S)} v Q	5 assoc
07.	{R v (S v P)} v Q	6 comm
08.	{(R v S) v P} v Q	7 assoc
09.	(R v S) v (P v Q)	8 assoc
10.	T v T	9, 1, 2 dil
11.	T	10 dup

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

01.	→ (P ⊃ Q) • (Q ⊃ R)	ACP
02.	P ⊃ Q	1 simp
03.	Q ⊃ R	1 simp
04.	~Q ⊃ ~P	2 contrap
05.	→ Q	ACP
06.	Q • Q	5 dup
07.	Q	6 simp
08.	Q ⊃ Q	5-7 CP
09.	~Q v Q	8 CE
10.	~P v R	3, 4, 9 dil
11.	P ⊃ R	3, 4, 9 dil
12.	{(P⊃Q)•(Q⊃R)}⊃(P⊃R)	1-11 CP

/ ∴ {(P • Q) ⊃ R} ≡ {P ⊃ (Q ⊃ R)}

01.	(P • Q) ⊃ R	ACP
02.	P	ACP
03.	Q	ACP
04.	~R	AIP
05.	~(P • Q)	4,1 MT
06.	P • Q	3,2 conj
07.	(P•Q)•~(P•Q)	6,5 conj
08.	R	4-7 IP
09.	Q ⊃ R	3-8 CP
10.	P ⊃ (Q ⊃ R)	2-9 CP
11.	{(P•Q)⊃R}⊃{P⊃(Q⊃R)}	1-10 CP
12.	P ⊃ (Q ⊃ R)	ACP

12.	P ⊃ (Q ⊃ R)	ACP
13.	~P v (Q ⊃ R)	12 CE
14.	~P v (~Q v R)	13 CE
15.	(~P v ~Q) v R	14 assoc
16.	~(P • Q) v R	15 DeM
17.	(P • Q) ⊃ R	16 CE
18.	{P⊃(Q⊃R)}⊃{(P•Q)⊃R}	12-17 CP
19.	[{P⊃(Q⊃R)}⊃{(P•Q)⊃R}] • [{(P•Q)⊃R} ⊃{P⊃(Q⊃R)}]	11-18conj
20.	{(P•Q)⊃R}≡{P⊃(Q⊃R)}	19 BE

01.	F	/ ∴ (G ⊃ H) v (~G ⊃ J)
02.	~(G ⊃ H)	ACP
03.	~(~G v H)	2 CE
04.	G • ~H	3 DeM
05.	G	4 simp
06.	G v J	5 DI
07.	~G ⊃ J	6 CE
08.	~(G ⊃ H) ⊃ (~G ⊃ J)	2-7 CP
09.	(G ⊃ H) v (~G ⊃ J)	8 CE

01.	F	/ ∴ (G ⊃ H) v (~G ⊃ J)
n-8.	~G v G	
n-7.	(~G v G) v (H v J)	n-8 DI
n-6.	{(~G v G) v H} v J	n-7 assoc
n-5.	{~G v (G v H)} v J	n-6 assoc
n-4.	{~G v (H v G)} v J	n-5 comm
n-3.	{(~G v H) v G} v J	n-4 assoc
n-2.	(~G v H) v (G v J)	n-3 assoc
n-1.	(~G v H) v (~G ⊃ J)	n-2 CE
n.	(G ⊃ H) v (~G ⊃ J)	n-1 CE

01.	F	/ ∴ (G ⊃ H) v (~G ⊃ J)
02.	G	ACP
03.	G • ~G	2 dup
04.	G	3 simp
05.	G ⊃ G	2-4 CP
06.	~G v G	5 CE
07.	(~G v G) v (H v J)	6 DI
08.	{(~G v G) v H} v J	7 assoc
09.	{~G v (G v H)} v J	8 assoc
10.	{~G v (H v G)} v J	9 comm
11.	{(~G v H) v G} v J	10 assoc
12.	(~G v H) v (G v J)	11 assoc
13.	(~G v H) v (~G ⊃ J)	12 CE
14.	(G ⊃ H) v (~G ⊃ J)	13 CE