Lecture 9
Conditional Proof, Nested Subproofs, & Tautologies

1. Conditional Proof
2. Examples
3. Nested Subproofs
4. Examples
5. Tautologies
6. Examples

Conditional Proof

Second of two 'proof strategies'

Basic idea: prove the conditional $\varphi \supset \psi$ is true, by assuming φ and deriving ψ.

Conditional Proof

At some point in a proof, you decide you'd like to be able to derive $\varphi \supset \psi$ on a line, but you can't figure out how. Add an assumption line consisting of φ, then proceed using the rules.

Conditional Proof

Keep deriving lines until you derive ψ. At this point, we don't know whether φ is actually true, since we just assumed it, but we have shown that if φ were true, then ψ would be true.

Conditional Proof

But this fact that the subproof demonstrated, that if φ is true, then ψ is true, just is what the conditional $\varphi \supset \psi$ means. So the subproof shows that the conditional can be validly infered.

Rules for the use of CP

1. **Start subproof (SP) by indenting and designating first line ACP**
2. ***CP ends any time you want***
3. **Mark off CP, closing SP and discharging assumption**
4. ***The next line after the closed CP SP can only be a conditional whose antecedent is the ACP and whose consequent is the last line of the CP SP***

Rules for the use of CP

5. All SPs must be closed before the proof can end
6. Once a SP has been closed, no lines in it may be used or cited

01.	G ⊃ T	
02.	(T v S) ⊃ K	/ ∴ G ⊃ K
03.	→ G	ACP
04.	T	1, 3 MP
05.	T v S	4 DI
06.	K	2, 5 MP
07.	G ⊃ K	3-6 CP

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP
04.	(A • D) v (A • E)	1, 2, 3 dil
05.	A • (D v E)	4 dist
06.	A	5 simp
07.	(C v B) ⊃ A	3-6 CP
08.	~(C v B) v A	7 CE

Nested Subproofs

So long as the rules for subproofs are followed, a single proof can have more than one subproof, and can even have subproofs within subproofs.

01.	~A ⊃ (B • C)	
02.	D ⊃ ~C	/ ∴ D ⊃ A
03.	→ D	ACP
04.	→ ~A	AIP
05.	~C	2, 3 MP
06.	B • C	1, 4 MP
07.	C	6 simp
08.	C • ~C	5, 7 conj
09.	A	4-8 IP
10.	D ⊃ A	3-9 CP

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP
04.	→ ~A	AIP
05.	(A • D) v (A • E)	1,2,3 dil
06.	A • (D v E)	5 dist
07.	A	6 simp
08.	A • ~A	5, 7 conj
09.	A	4-8 IP
10.	(C v B) ⊃ A	3-9 CP
11.	~(C v B) v A	10 CE

01. (~M v P) ⊃ (K • ~L)
02. ~K v L / ∴ M v K
03. → ~M ACP
04. → ~K AIP
05. ~M v P 3 DI
06. K • ~ L 5, 1 MP
07. K 6 simp
08. K • ~K 4, 7 conj
09. K 4-8 IP
10. ~M ⊃ K 3-9 CP
11. M v K 10 CE

Tautologies

The proof method is a method for demonstrating validity, for demonstrating that *if* a given set of statements (premises) is true, then another statement (the conclusion) is true. Given this, is there any way we could use the proof method to show that a statement is a tautology?

Tautologies

Yes. If a statement can be derived from *no* premises, then we know that that statement follows from anything, it will follow from any set of premises. (If you can derive it from no premises, then clearly if you had premises, regardless of what those premises were, you would also be able to derive it.)

Tautologies

This means that any argument that has this statement as its conclusion is valid. And if this is true, then the statement must be a tautology. No statement of any other category could possibly have this property.

/ ∴ (P ⊃ ~P) ⊃ ~P
1. → P ⊃ ~P ACP
2. ~P v ~P 1 CE
3. ~P 2 dup
4. (P ⊃ ~P) ⊃ ~P 1-3 CP

/ ∴ ~P ⊃ (P ⊃ ~P)
1. → ~P ACP
2. ~P v ~P 1 dup
3. P ⊃ ~P 2 CE
4. ~P ⊃ (P ⊃ ~P) 1-3 CP

/ ∴ ~(A v B) v ~(~A • ~B)

1.	A v B	ACP
2.	~A • ~B	AIP
3.	~A	2 simp
4.	~B	2 simp
5.	B	1, 3 DS
6.	B • ~B	4, 5 conj
7.	~(~A • ~B)	2-6 IP
8.	(A v B) ⊃ ~(~A • ~B)	1-7 CP
9.	~(A v B) v ~(~A • ~B)	8 CE