Lecture 08
Proofs: Replacement Rules & Indirect Proof

1. **Last 7 replacement rules: DeM, BE, contra, dist, exp, assoc, dup**
2. **Examples**
3. **Introduction to Indirect Proof**
4. **Examples**

Last Seven Replacement Rules

DeMorgan's (DeM)

$$\sim(\varphi \vee \psi) \;::\; \sim\varphi \bullet \sim\psi$$

$$\sim(\varphi \bullet \psi) \;::\; \sim\varphi \vee \sim\psi$$

Last Seven Replacement Rules

Biconditional Exchange (BE)

$$\varphi \equiv \psi \;::\; (\varphi \supset \psi) \bullet (\psi \supset \varphi)$$

Last Seven Replacement Rules

Contraposition (contra)

$$\varphi \supset \psi \;::\; \sim\psi \supset \sim\varphi$$

Last Seven Replacement Rules

Distribution (dist)

$$\varphi \bullet (\psi \vee \chi) \;::\; (\varphi \bullet \psi) \vee (\varphi \bullet \chi)$$

$$\varphi \vee (\psi \bullet \chi) \;::\; (\varphi \vee \psi) \bullet (\varphi \vee \chi)$$

Last Seven Replacement Rules

Exportation (exp)

$$\varphi \supset (\psi \supset \chi) \;::\; (\varphi \bullet \psi) \supset \chi$$

Last Seven Replacement Rules

Association (assoc)

$(\varphi \vee \psi) \vee \chi \;::\; \varphi \vee (\psi \vee \chi)$

$(\varphi \bullet \psi) \bullet \chi \;::\; \varphi \bullet (\psi \bullet \chi)$

Last Seven Replacement Rules

Duplication (dup)

$\varphi \;::\; \varphi \vee \varphi$

$\varphi \;::\; \varphi \bullet \varphi$

1. A	/ ∴ ~(~A • ~B) ⊃ ~P	
2. ~(P ⊃ ~P) ⊃ (~A • ~B)		
3. A v B		1 DI
4. ~(~A • ~B)		3 DeM
5. ~(~A • ~B) ⊃ (P ⊃ ~P)		2 contra
6. ~(~A • ~B) ⊃ (~P v ~P)		5 CE
7. ~(~A • ~B) ⊃ ~P		6 dup

01. (I v S) ⊃ C		
02. D ⊃ (P • I)	/ ∴ D ⊃ C	
03. ~D v (P • I)		2 CE
04. (~D v P) • (~D v I)		3 dist
05. ~D v I		4 simp
06. D ⊃ I		5 CE
07. ~C ⊃ ~(I v S)		1 contra
08. C v ~(I v S)		7 CE
09. C v (~I • ~S)		8 DeM
10. (C v ~I) • (C v ~S)		9 dist
11. C v ~I		10 simp
12. ~I v C		11 comm
13. I ⊃ C		12 CE
14. D ⊃ C		6, 13 HS

Indirect Proof

One of two 'proof strategies', the other is *conditional proof*

Basic idea: prove that φ is true, not by deriving φ directly, but by showing that ~φ must be false.

Indirect Proof

At some point in a proof, you decide you'd like to be able to derive φ on a line, but you can't figure out how. Add an assumption line consisting of ~φ, then proceed using the rules.

Indirect Proof

Keep deriving lines until you derive an *explicit contradiction*. We know that contradictions are always false. But we also know that our rules are truth preserving, and so if they are applied to only true statements they will produce only true statements.

Indirect Proof

But: we managed to produce a false statement, the explicit contradiction. So, the set of statements we were applying the rules to must not all have been true.

Indirect Proof

But the only statement that is suspect is the one we added as an assumption: ~φ. So *that one* must be the one that is false. And if ~φ is false, then φ must be true. So you are justified in writing a new derived line consisting of φ.

1.	A ⊃ B	
2.	B ⊃ ~B	/ ∴ ~A
3.	~B v ~B	2 CE
4.	~B	3 dup
5.	~A	4, 1 MT

1.	A ⊃ B	
2.	B ⊃ ~B	/ ∴ ~A
3.	A	AIP
4.	B	1, 3 MP
5.	~B	4, 2 MP
6.	B • ~B	4, 5 conj
7.	~A	3-6 IP

01.	(~M v P) ⊃ (K • ~L)	
02.	~K v L	/ ∴ M v K
03.	~M	AIP
04.	~M v P	3 DI
05.	K • ~L	1, 4 MP
06.	K	5 simp
07.	~L	5 simp
08.	~K	7, 2 DS
09.	K • ~K	6, 8 conj
10.	M	3-9 IP
11.	M v K	10 DI

01.	(I v S) ⊃ C	
02.	D ⊃ (P • I)	/ ∴ D ⊃ C
03.	~(D ⊃ C)	AIP
04.	~(~D v C)	3 CE
05.	D • ~C	4 DeM
06.	D	5 simp
07.	~C	5 simp
08.	P • I	6, 2 MP
09.	I	8 simp
10.	I v S	9 DI
11.	C	1, 10 MP
12.	C • ~C	7, 11 conj
13.	D ⊃ C	3-12 IP

Rules for the use of IP

1. Start subproof (SP) by indenting and designating first line AIP
2. IP ends only when an explicit contradiction is derived
3. Mark off IP, closing SP and discharging assumption
4. The next line after the closed IP SP can only be the negation of the AIP

Rules for the use of IP

5. All SPs must be closed before the proof can end
6. Once a SP has been closed, no lines in it may be used or cited