Lecture 07
Proofs: Inference & Replacement Rules

1. **Last 3 inference rules: Dil, DI, conj**

2. **Examples**

3. **First 3 replacement rules: CE, DN, comm**

4. **Examples**

Final 3 Inference Rules

Dilemma (dil)

$$\varphi \supset \psi$$
$$\chi \supset \xi$$
$$\varphi \lor \chi$$
$$\therefore \psi \lor \xi$$

Final 3 Inference Rules

Disjunction Introduction (DI)

$$\varphi$$
$$\therefore \varphi \lor \psi$$

Final 3 Inference Rules

Conjunction (conj)

$$\varphi$$
$$\psi$$
$$\therefore \varphi \bullet \psi$$

---

1. (D ∨ G) ⊃ (A ∨ C)	
2. D • (A ⊃ G)	
3. C ⊃ H	/ ∴ D • (H ∨ G)
4. D	2 simp
5. D ∨ G	4, DI
6. A ∨ C	1, 5 MP
7. A ⊃ G	2 simp
8. H ∨ G	3, 6, 7 dil
9. D • (H ∨ G)	4, 8 conj

---

1. (~A ⊃ ~C) ⊃ A	
2. C ⊃ (~A ⊃ ~C)	
3. ~A	/ ∴ ~C
4. ~(~A ⊃ ~C)	3, 1 MT
5. ~C	2, 4 MT
4. C ⊃ A	1, 2 HS
5. ~C	3, 4 MT

Card 1

1. (R v G) ⊃ ~A
2. A v W
3. R • S   / ∴ W v X

4. R   3 simp
5. R v G   4 DI
6. ~A   1, 5 MP
7. W   6, 2 DS
8. W v X   7 DI

Card 2

1. M ⊃ D
2. G ⊃ M
3. G v G   / ∴ D v M

4. G ⊃ D   1, 2 HS
5. D v M   2, 3, 4 dil

Card 3

First Three Replacement Rules

Conditional Exchange (CE)

$$\varphi \supset \psi \;::\; {\sim}\varphi \vee \psi$$

Card 4

First Three Replacement Rules

Double Negation (DN)

$$\varphi \;::\; {\sim}{\sim}\varphi$$

Card 5

First Three Replacement Rules

Commutation (comm)

$$\varphi \vee \psi \;::\; \psi \vee \varphi$$

$$\varphi \bullet \psi \;::\; \psi \bullet \varphi$$

Card 6

Differences between Inference Rules and replacement Rules

Can be used on *part* of a line

x. A ⊃ (B ⊃ C)
y. A ⊃ (~B v C)   x CE
z. ~A v (~B v C)   y CE

Differences between Inference Rules and replacement Rules

Can go in *either* direction

  x. ~A ∨ (~B ∨ C)

  y. A ⊃ (~B ∨ C)   x CE

  z. A ⊃ (B ⊃ C)   y CE

---

1. C ∨ (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R   / ∴ ~~R
4. ~C ⊃ (D • E)   1 CE
5. A ⊃ (D • E)   2, 4 HS
6. ~~A ⊃ (D • E)   5 DN
7. ~~A ⊃ (E • D)   6 comm
8. R   3, 7 MP
9. ~~R   8 DN

---

1. A ∨ B
2. C ∨ D
3. B ⊃ ~D   / ∴ ~A ⊃ ~~C
4. ~A ⊃ B   1 CE
5. D ∨ C   2 comm
6. ~D ⊃ C   5 CE
7. ~A ⊃ ~D   3, 4 HS
8. ~A ⊃ C   6, 7 HS
9. ~A ⊃ ~~C   8 DN

---

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q)   / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P ∨ Q) ⊃ K   1 CE
4. (Q ∨ ~P) ⊃ K   3 comm
5. (~Q ⊃ ~P) ⊃ K   4 CE
6. (~Q ⊃ ~P) ⊃ (P • Q)   5, 2 HS
7. (~Q ⊃ ~P) ⊃ (Q • P)   6 comm