Lecture 06
Proofs: Inference Rules

1. Introduction to the proof method

2. Two inference rules: MP, MT, examples

3. Discussion

4. DS, HS, simp, examples

Introduction to the Proof Method

1. Representation of argument

1. (A • B) ⊃ C

2. A • B / ∴ C

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)	
2. ~D • G	**/ ∴ ~E**
3. ~D	**2 simp**
4. E ⊃ ~G	**3, 1 DS**
5. G	**2 simp**
6. ~E	**5, 4 MT**

Introduction to the Proof Method

3. Why does it work?

Rules are Truth Preserving

If they are applied to only true lines, they will produce only true lines

First two Inference Rules

Modus Ponens (MP)

$\varphi \supset \psi$
φ
$\therefore \psi$

First two Inference Rules

Modus Tolens (MT)

$\varphi \supset \psi$
$\sim\psi$
$\therefore \sim\varphi$

1. (A • B) ⊃ C

2. A • B / ∴ C

3. C 1, 2 MP

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K

5. ~P 1, 4 MT
6. A ≡ K 3, 5 MP

Three more Inference Rules

Disjunctive Syllogism (DS)

$\varphi \vee \psi$
$\sim\varphi$
$\therefore \psi$

Three more Inference Rules

Hypothetical Syllogism (HS)

$\varphi \supset \psi$
$\psi \supset \chi$
$\therefore \varphi \supset \chi$

Three more Inference Rules

Simplification (simp)

$\varphi \bullet \psi$
$\therefore \psi$

1. D ∨ (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp
4. ~D 2 simp
5. E ⊃ ~G 4, 1 DS
6. ~E 3, 5 MT

1. (B ⊃ A) • (C ⊃ D)	
2. ~B	
3. B v ~K	
4. ~(A ⊃ D) ⊃ K	**/ ∴ B ⊃ D**
5. ~K	**2, 3 DS**
6. A ⊃ D	**5, 4 MT**
7. B ⊃ A	**1 simp**
8. B ⊃ D	**6, 7 HS**

1. ~D v (H v H)	
2. ~R • (H ⊃ ~ J)	
3. (R • D) ⊃ ~J	
4. (H ⊃ ~ J) ⊃ (H ⊃ R)	
5. D	**/ ∴ ~J**
6. H ⊃ ~J	**2 simp**
7. H v H	**5, 1 DS**
8. H ⊃ R	**6, 4 MP**
9. ~R	**2 simp**
10. ~H	**9, 8 MT**
11. H	**10, 7 DS**
12. ~J	**6, 11 MP**

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	**/ ∴ ~B**
6. R ⊃ R	**2, 4 HS**
7. K	**6, 1 DS**
8. S • A	**7, 3 MT**
9. A	**8 simp**
10. B ⊃ ~K	**9, 5 MP**
11. ~B	**10, 7 MT**