Lecture 5: Equivalence, consistency, implication, & validity

1. Relations between two statements: equivalence, consistency, implication

2. Relations between three or more statements: equivalence, consistency, joint implication

3. Argument validity

Relations between two statements

Just as it is often helpful to know whether an individual statement is a tautology, contradiction, or contingency, it is also often helpful to know what relations hold between two statements.

Equivalence

Two statements are equivalent iff they have identical truth columns.

To test for equivalence, construct a joint truth table for the two statements and compare their truth columns. If the columns are identical, then the statements are equivalent. If they are not identical, then they are not equivalent.

Consistency

Two statements are consistent iff it is possible for them to both be true at the same time.

To test for consistency, do a joint truth table for the two statements. If there is a row (one or more) on which both statements are T, then they are consistent. If

Implication

Statement φ implies statement ψ iff:

if φ is true, then ψ must be true.

To see whether φ implies ψ, do a joint truth table for φ and ψ, and look for a row on which φ is T and ψ is F (a *counter-example row*). If there IS a counter-example row, then then φ does NOT imply ψ; if there is NOT a counter-example row, then φ DOES imply ψ.

1. ~(P • Q) **2. ~P • ~Q**

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Are (1) and (2) equivalent? No

1. ~(P • Q) 2. ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Are (1) and (2) consistent? Yes

1. ~(P • Q) 2. ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (1) imply (2)? No

1. ~(P • Q) 2. ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (2) imply (1)? Yes

1. A ≡ B 2. (A ⊃ B) • (B ⊃ A)

A	B	A ⊃ B	B ⊃ A	A ≡ B	(A ⊃ B) • (B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Are (1) and (2) equivalent? Yes

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A	B	A ⊃ B	B ⊃ A	A ≡ B	(A ⊃ B) • (B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Are (1) and (2) consistent? Yes

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A	B	A ⊃ B	B ⊃ A	A ≡ B	(A ⊃ B) • (B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Does (1) imply (2)? Yes

1. A ≡ B 2. (A ⊃ B) • (B ⊃ A)

A	B	A ⊃ B	B ⊃ A	A ≡ B	(A ⊃ B) • (B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Does (2) imply (1)? Yes

1. (M ⊃ N) • ~N 2. ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Are (1) and (2) equivalent? No

1. (M ⊃ N) • ~N 2. ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Are (1) and (2) consistent? Yes

1. (M ⊃ N) • ~N 2. ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (1) imply (2)? Yes

1. (M ⊃ N) • ~N 2. ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (2) imply (1)? No

1. S • (J v K) 2. (S • J) v (S • K)

J	K	S	J**v**K	S•J	S•K	S•(J**v**K)	(S•J)**v**(S•K)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Are (1) and (2) equivalent? Yes

1. S • (J v K) 2. (S • J) v (S • K)

J	K	S	J**v**K	S•J	S•K	S•(J**v**K)	(S•J)**v**(S•K)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Are (1) and (2) consistent? Yes

1. S • (J v K) 2. (S • J) v (S • K)

J	K	S	J**v**K	S•J	S•K	S•(J**v**K)	(S•J)**v**(S•K)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Does (1) imply (2)? Yes

1. S • (J v K) 2. (S • J) v (S • K)

J	K	S	J**v**K	S•J	S•K	S•(J**v**K)	(S•J)**v**(S•K)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Does (2) imply (1)? Yes

Relations between three or more statements

Equivalence

A set of statements are equivalent if they all have identical truth columns

Consistency

A set of statements are consistent if there is at least one row on which they are all T.

Joint Implication

A set of statements ($\varphi_1 \ldots \varphi_n$) jointly imply statement ψ iff: if ($\varphi_1 \ldots \varphi_n$) are all true, then ψ must be true.

To test whether statements ($\varphi_1 \ldots \varphi_n$) jointly imply statement ψ, do a joint truth table with all statements, and look for a row in which ALL of ($\varphi_1 \ldots \varphi_n$) are T and ψ is F (a CE row). If there IS such a row, then ($\varphi_1 \ldots \varphi_n$) do NOT jointly imply ψ; if there is NOT such a row, then ($\varphi_1 \ldots \varphi_n$) DO jointly imply ψ.

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G v H	~H ⊃ ~G	~G v H
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	F	T	T

Are 1-3 equivalent? Yes

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G v H	~H ⊃ ~G	~G v H
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	F	T	T

Are 1-3 consistent? Yes

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G v H	~H ⊃ ~G	~G v H
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	F	T	T

Do 1-2 jointly imply 3? Yes

1. L
2. L ⊃ K
3. ~K

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 equivalent? No

1. L
2. L ⊃ K
3. ~K

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 consistent? No

1. L
2. L ⊃ K
3. ~K

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1-2 jointly imply 3? No

1. L
2. L ⊃ K
3. ~K

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1&3 jointly imply 2? No

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 equivalent? No

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 consistent? Yes

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1-3 jointly imply 4? Yes

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1, 2 & 4 jointly imply 3? No

Argument validity

An argument is deductively valid iff the premises jointly imply the conclusion.

1. P v C
2. ~C
∴ 3. P

C	P	~C	P v C
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

This pizza is either pepperoni or cheese. It is not cheese. Therefore, it is pepperoni.

Argument is valid

1. G
2. L
∴ 3. D

D	G	L
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Trees are green. Lead is heavy. Therefore, Elvis is dead.

Argument is invalid

1. D ⊃ R
2. D
∴ 3. R

D	R	D ⊃ R
T	T	T
T	F	F
F	T	T
F	F	T

If ducks sink, then ducks are made of small rocks. Ducks do sink. Therefore, ducks are made of small rocks.

Argument is valid

1. ~A ⊃ K
2. ~K ≡ ~A
3. ~K
∴ 4. J

Argument is valid

A	J	K	~A	~K	~A ⊃ K	~K ≡ ~A
T	T	T	F	F	T	T
T	T	F	F	T	T	F
T	F	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	T