Lecture 3: Translating from spoken language to formal notation

1. Translating five statement operators
2. Translating compound statements
3. Translating arguments

Five Statement Operators: Negation

Negation is expressed in a number of ways in English

The cat is on the mat.

The cat is *not* on the mat.

The cat is*n't* on the mat.

***It is false that* the cat is on the mat.**

Five Statement Operators: Negation

The cat is on the mat. = C

The cat is *not* on the mat.

The cat is*n't* on the mat.

***It is false that* the cat is on the mat.**

~C

Five Statement Operators: Conjunction

Roses are red. = R

Violets are blue. = V

Roses are red and violets are blue.

Both roses are red and violets are blue.

Roses are red but violets are blue.

R • V

Five Statement Operators: Conjunction

Roses are red. = R

Apples are red. = A

Roses are red and apples are red.

Both roses and apples are red.

Roses and apples are red.

R • A

Five Statement Operators: Conjunction

Apples are sweet. = S

Apples are red. = A

Apples are sweet and apples are red.

Apples are both sweet and red.

Apples are sweet and red.

S • A

Five Statement Operators: Conjunction

Components of a conjunction are *conjuncts*.

S • A

Five Statement Operators: Disjunction

Dallas will win the Superbowl.

Buffalo will win the Superbowl.

Dallas will win the Superbowl or Buffalo will win the Superbowl.

Either Dallas will win the Superbowl or Buffalo will win the Superbowl.

Five Statement Operators: Disjunction

Either Dallas will win the Superbowl or Buffalo will win the Superbowl.

Either Dallas or Buffalo will win the Superbowl.

Dallas will win the Superbowl *unless* Buffalo wins the Superbowl.

Five Statement Operators: Disjunction

Dallas will win the Superbowl. = D

Buffalo will win the Superbowl. = B

Dallas will win the Superbowl or Buffalo will win the Superbowl.

D v B

Five Statement Operators: Disjunction

Components of a disjunction are called *disjuncts*.

D v B

Five Statement Operators: Conditional

You earn exactly 900 points. = N

You get a some form of A. = A

***If* you earn exactly 900 points, *then* you will get some form of A.**

N ⊃ A

Statement following 'if' is antecedent, statement following 'then' is consequent. (Does not apply to '*only* if'.)

Five Statement Operators: Conditional

If **you earn exactly 900 points, you will get some form of A.**

N ⊃ A

Statement following 'if' is antecedent, statement following where the 'then' *would be* is consequent.

Five Statement Operators: Conditional

Statement following 'if' is antecedent, statement following where the 'then' *would be* is consequent.

Statement following 'if' is antecedent, statement following the 'then' is consequent.

Notice: *Order in which they appear is irrelevant!*

Five Statement Operators: Conditional

If **you earn exactly 900 points, you will get some form of A.**

You will get some form of A if you earn exactly 900 points.

N ⊃ A

Five Statement Operators: Conditional

In English, which statement is the antecedent and which the consequent is not coded by linear order, but by words in sentence, like 'if', 'then', 'only if', and others. So to tell which is the antecedent and which the consequent, ignore the order, look for key words.

Five Statement Operators: Conditional

In our logical notation, the antecedent and conditional are coded by linear order: *antecedent* is always on the left, *consequent* always on the right.

N ⊃ A

Five Statement Operators: Conditional

Jim will go to Hawaii. = H

Jim wins Lotto. = L

Jim will go to Hawaii if Jim wins Lotto.

L ⊃ H

Jim will go to Hawaii only if Jim wins Lotto.

Five Statement Operators: Conditional

The statement following 'only if' is the *consequent*.

Jim will go to Hawaii only if Jim wins Lotto.

H ⊃ L

Five Statement Operators: Conditional

For conditionals using 'if':

- If 'if' is by itself (not immediately preceded by 'only), then statement following the 'if' is the antecedent, the other statement (may, or may not, have a 'then') is the consequent.

Five Statement Operators: Conditional

For conditionals using 'if':

- If the expression is 'only if', then statement following the 'only if' is the consequent, the other statement is the antecedent.

Five Statement Operators: Conditional

A sufficient condition is a condition that suffices for something else (but it may not be necessary).

-Earning 800 points is sufficient for passing the class, but it is not necessary.

-Melting my car into slag is sufficient to render it undrivable, but it is not *necessary*.

Five Statement Operators: Conditional

A necessary condition is a condition that is required for something else (but it may not be sufficient).

-Hydrogen is necessary for water (but it is not sufficient)

- Passing the qualifying exam is necessary for getting a PhD (but it is not sufficient)

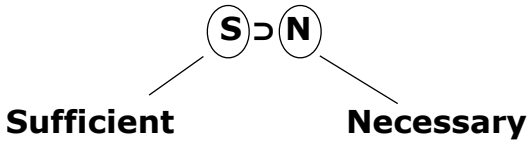
Five Statement Operators: Conditional

For conditionals using 'necessary' or 'sufficient':

- The statement identified as the necessary condition is the consequent

- the statement identified as the sufficient condition is the antecedent

Five Statement Operators: Conditional



Five Statement Operators: Conditional

… is sufficient for …

Getting a C- or better is sufficient for getting a P.

You earning a C- or better is sufficient for you getting a P.

Five Statement Operators: Conditional

You earning a C- or better is sufficient for you getting a P.

You earn a C- or better. = C
You get a P. = P

C ⊃ P

Five Statement Operators: Conditional

You earning a C- or better is necessary for you getting a P.

You earn a C- or better. = C
You get a P. = P

P ⊃ C

Five Statement Operators: Conditional

It is necessary for … that …

It is necessary for you to get a driver's license that you pass the written exam.

You get a driver's license. = L
You pass the written exam. = E

Five Statement Operators: Conditional

You get a driver's license. = L

You pass the written exam. = E

It is necessary for L that E.

L ⊃ E

Five Statement Operators: Biconditional

… if and only if …

… iff …

… is necessary and sufficient for …

Five Statement Operators: Biconditional

You will get a P in the class if and only if you earn a C- or better.

You get a P. = P
You earn a C- or better. = C

P ≡ C

No special name for components of a biconditional

Translating compound statements

If Sarah doesn't come to the party, then we won't need the vegetarian burgers.

S = Sarah comes to the party
V = We will need the vegetarian burgers.
If not-S, then not-V.

~S ⊃ ~V

Translating compound statements

I'll buy an Xbox 360, and either a PSP or Nintendo DS

X = I'll buy an Xbox 360
P = I'll buy a PSP
N = I'll buy a Nintendo DS

X, and either P or N
X, and (either P or N)

X • (P v N)

Translating compound statements

I'll buy an Xbox 360 and a PSP or Nintendo DS

X = I'll buy an Xbox 360
P = I'll buy a PSP
N = I'll buy a Nintendo DS

X and P or N

(X and P) or N X and (P or N)

5 x 4 + 1

Translating compound statements

The car runs well and the motorcycle runs well, but you should be careful if you take the boat out.

C = The car runs well
M = The motorcycle runs well
Y = You should be careful
B = You take the boat out

Translating compound statements

The car runs well and the motorcycle runs well, but you should be careful if you take the boat out.

C and M, but Y if B

(C and M), but (Y if B)

(C • M), • (Y if B)

(C • M) • (B ⊃ Y)

Translating compound statements

Either you will get a fine or you will have to do hard time, if you don't pay your taxes and you don't file an extension.

F = You will get a fine
H = You will do hard time
T = You pay your taxes
E = You file an extension

Either you will get a fine or you will have to do hard time, if you don't pay your taxes and you don't file an extension.

F = You will get a fine
H = You will do hard time
T = You pay your taxes
E = You file an extension

Either F or H, if not-T and not-E
(Either F or H), if (~T and ~E)
(F v H), if (~T • ~E)
(~T • ~E) ⊃ (F v H)

It is necessary for you to graduate with honors both that you have at least a 3.5 GPA, and you took classes P/NP only if the classes were not in your major.

H = You graduate with honors
G = You have at least a 3.5 GPA
P = You took classes P/NP
M = The classes were in your major

It is necessary for H both that G, and P only if not-M.

It is necessary for H both that G, and P only if not-M.

~~**(It is necessary for H both that G), and (P only if not-M).**~~

~~**(<u>It is necessary for H both that G, and P</u>) only if (*not-M*)**~~

It is necessary for H {both that G, and (P only if not-M)}.

It is necessary for H {both that G, and (P only if ~M)}

It is necessary for H {both that G, and (P ⊃ ~M)}

It is necessary for H that {G • (P ⊃ ~M)}

H ⊃ {G • (P ⊃ ~M)}

Translating arguments

An argument is a set of statements, so to translate an argument, you just translate all the statements, indicating which is the conclusion by placing it at the end with a triple-dot '∴'.

Either you were at work, or you were at the bar. But your office wasn't open today today. Therefore, you were at the bar.

1. Either you were at work, or you were at the bar.

2. But your office wasn't open today.

∴ 3. You were at the bar.

1. Either you were at work, or you were at the bar.

2. But your office wasn't open today.

∴ 3. You were at the bar.

W = You were at work
B = You were at work
O = Your office was open today

1. Either W, or B	**1. W v B**
2. But not-O.	**2. O**
∴ 3. B.	**∴ 3. ~B**

If Moe slaps Larry upside the head, then Larry will slap Curley upside the head. Curley will hit Moe with a hammer only if Larry hits slaps him (Curley) upside the head. It is sufficient for Curley to hit Moe with a hammer that Moe does not slap Larry upside the head. But Moe does slap Larry upside the head. Therefore, Curley will hit Moe with a hammer if Larry slaps Curley upside the head.

M = Moe slaps Larry upside the head
L = Larry slaps Curley upside the head.
C = Curley will hit Moe with a hammer

If Moe slaps Larry upside the head, then Larry will slap Curley upside the head. Curley will hit Moe with a hammer only if Larry hits slaps him (Curley) upside the head. It is sufficient for Curley to hit Moe with a hammer that Moe does not slap Larry upside the head. But Moe does slap Larry upside the head. Therefore, Curley will hit Moe with a hammer if Larry slaps Curley upside the head.

If M, then L.
C only if L.
It is sufficient for C that not-M.
But M.
Therefore, C if L.

If M, then L.
C only if L.
It is sufficient for C that not-M.
But M.
Therefore, C if L.

1. If M, then L
2. C only if L.
3. It is sufficient for C that not-M.
4. But M.
Therefore 5. C if L.

1. If M, then L
2. C only if L.
3. It is sufficient for C that not-M.
4. But M.
Therefore 5. C if L.

1. M ⊃ L
2. C ⊃ L
3. ~M ⊃ C
4. M
∴ 5. L ⊃ C