Lecture 2: Arguments, Statements, and Recursion

Arguments

Arguments are verbal, written, propositional, representations of episodes of reasoning

Reasoning is determining what follows from facts or assumptions

An argument: A set of statements such that one (conclusion) is taken to follow from the others (premises)

1. Either the pizza in my hand is a cheese pizza or the pizza in my hand is a pepperoni pizza.

2. The pizza in my hand is not a pepperoni pizza.

∴ 3. Therefore, the pizza in my hand is a cheese pizza.

Will always be exactly 1 conclusion

Can be any number of premises

Varieties of arguments

Two kinds of argument

1. Deductive - Premises are taken to provide complete, watertight support for the conclusion (may or may not be successful)

2. Inductive - Premises are taken to provide probable support for the conclusion, but not watertight support (may or may not be successful)

Example of *deductive* argument

1. If I file my taxes I will get a refund.

2. I will file my taxes.

∴ 3. I will get a refund.

Example of *deductive* argument

1. If John wins Lotto, then John will go to Hawaii.

2. John went to Hawaii.

∴ 3. John won Lotto.

Example of *inductive* argument

1. Southpark has always been on Wednesday at 10 pm.

2. It is now Wednesday at 10 pm.

∴ 3. Therefore, Southpark is [probably] on now.

Example of *inductive* argument

1. The last time I watched Southpark, the first commercial was for Old Navy.

∴ 2. The first commercial on tonight's episode of Southpark will [probably] be for Old Navy.

Difference between deductive and inductive arguments is *not* a matter of how good the arguments are. There are good and bad inductive arguments, and good and bad deductive arguments.

There are *two distinct measures* of an argument's goodness:

1. The 'inferential relationship' between the premises and the conclusion

2. The truth of the premises

1. The 'inferential relationship' between the premises and the conclusion

If the premises *were* true, would the conclusion necessarily (deductive), or probably (inductive), be true?

Note: This can be assessed even if the premises are *in fact* false.

Example of *deductive* argument

1. If I file my taxes I will get a refund.

2. I will file my taxes.

∴ 3. I will get a refund.

If these premises were/are true, then the conclusion would have to be true.

If the conclusion is/were false, then at least one of the premises would have to be false.

Example of *deductive* argument

1. If John wins Lotto, then John will go to Hawaii.

2. John went to Hawaii.

∴ 3. John won Lotto.

Even if these premises were/are true, the conclusion could be false.

If a *deductive* argument has a good inferential relationship between the premises and conclusion, then it is *valid*.

An argument (deductive) is valid iff:

-If all the premises of the argument were true, the conclusion would have to be true

-If the conclusion is false, then one or more of the premises must be false

A valid deductive argument with false premises and a false conclusion:

1. If I am President of the US, then I get all the free BMWs I want.

2. I am President of the US.

∴ 3. I get all the free BMWs I want.

An *invalid* deductive argument with *true* premises and a *true* conclusion:

1. If I have a mass greater than 0 kg, then I am subject to gravitational forces.

2. I like pizza.

∴ 3. Paris is in France.

Example of *inductive* argument

1. Southpark has always been on Wednesday at 10 pm.

2. It is now Wednesday at 10 pm.

∴ 3. Therefore, Southpark is [probably] on now.

In this argument, there is a good inferential relation between premises and conclusion.

Example of *inductive* argument

1. The last time I watched Southpark, the first commercial was for Old Navy.

∴ 2. The first commercial on tonight's episode of Southpark will [probably] be for Old Navy.

In this argument, there is *not* a good inferential relation between premises and conclusion.

If an *inductive* argument has a good inferential relationship between the premises and conclusion, then it is *strong*.

An argument (inductive) is strong iff:

-If all the premises of the argument were true, the conclusion would probably be true

-If the conclusion is false, then one or more of the premises is probably false

First kind of goodness that an argument can have: an appropriate inferential relation between the premises and the conclusion.

If a deductive argument has it, it is *valid*.

If an inductive argument has it, it is *strong*.

Second kind of goodness an argument can have is the truth of its premises

1. If I am President of the US, then I get all the free BMWs I want.

2. I am President of the US.

∴ 3. I get all the free BMWs I want.

1. If I am over 21, then I can legally drink beer at the Pub.

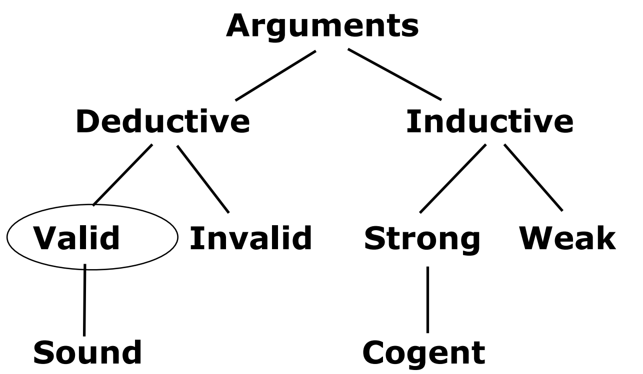
2. I am over 21.

∴ 3. I can legally drink beer at the Pub.

Second kind of goodness an argument can have is the truth of its premises

If a deductive argument (i) is valid, and (ii) has all true premises, then it is *sound*.

If an inductive argument (i) is strong, and (ii) has all true premises, then it is *cogent*.



Validity

Validity is often a function of the formal features of an argument

Either the pizza in my hand is a cheese pizza or it is a pepperoni pizza.

The pizza in my hand is not a pepperoni pizza.

Therefore, it is a cheese pizza.

Either X or Y.

Not Y.

Therefore X.

Validity

Validity is often a function of the formal features of an argument

Either the rabbit ran down the left trail or the rabbit ran down the right trail.

The rabbit did not run down the right trail.

Therefore, the rabbit ran down the left trail.

Either X or Y.

Not Y.

Therefore X.

So we will:

1. develop means to determine an argument's form

2. develop tools for determining, of a given formal representation of an argument, whether or not is it valid

Argument form

An argument is a set of statements, and accordingly an argument's logical form is determined by the logical form of the statements (the premises and conclusion).

A statement's logical form (at least those we will discuss in this course) is determined by the simple statements it has as components, and the operators (if any) that combine those components.

Statements:
Simple and Compound

A statement is something that makes a claim: typically expressed with a declarative sentence (not a question, exclamation, imperative, etc.)

A simple statement is a statement that *does not* have parts that are themselves statements.

A compound statement is a statement that *does* have parts that are themselves statements.

1. Either <u>the rabbit ran down the left trail</u> or <u>the rabbit ran down the right trail</u>.

2. The rabbit did not run down the right trail.

∴ 3. The rabbit ran down the left trail.

1. Either <u>the rabbit ran down the left trail</u> or <u>the rabbit ran down the right trail</u>.

2. It is false that <u>the rabbit ran down the right trail.</u>

∴ 3. The rabbit ran down the left trail.

Two simple statements:

The rabbit ran down the left trail.

The rabbit ran down the right trail.

Compound statements

Compound statements are built up from simple statements with statement operators

Either … or …

It is false that …

Five statement operators (aka logical operators)

Negation: It is false that … ; …n't

Conjunction: … and …

Disjunction: … or …

Conditional: if … then …

Biconditional: … if and only if …

John is at the party.

There is pizza at the party.

John isn't at the party.

There is pizza at the party, and John is at the party.

Either John is at the party, or there is pizza at the party.

If there is pizza at the party, then John is at the party.

Recursion

The five statement operators are *recursive*, meaning that they can take as input not only simple statements, but can also take as inputs compound statements formed by statement operators.

You pay me the $20 you owe me.
You come to the party.
I'll jack your ride.
I'll steal your wallet.

If you come to the party, then I'll steal your wallet.

If <u>you come to the party and you don't pay me the $20 you owe me</u>, then <u>either I'll steal your wallet or I'll jack your ride</u>.

Arithmetic operators

Four arithmetic operators

Addition
Subtraction
Multiplication
Division

Arithmetic operators

Arithmetic operators form complex arithmetic expressions from simple numerical expressions (numerals)

5 + 3 **5 - 3**
5 x 3 **5 ÷ 3**

Arithmetic operators

Arithmetic operators are *recursive*, in that they can also operate on expressions that are themselves the product of an arithmetic operator:

5 + 3
(6 - 2) + (4 x 9)
{(8 x 8) ÷ (4 - 1)} + (3 - 1)